Spin-2 dark matter from inflation

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Outline

1. Dark matter from spectator fields during inflation
2. Spin of dark matter: Spin-0, spin-1, spin-2
3. Summary

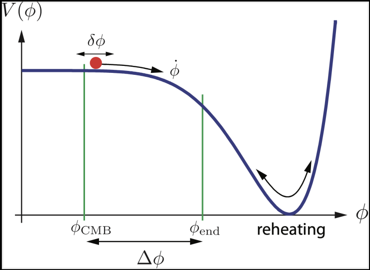
Slow-roll inflation

Scalar field with slow-roll potentials

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2}\mathrm{R} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)$$

$$\epsilon = -\frac{\dot{H}}{H^2} \simeq \frac{M_{\rm Pl}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2 \ll 1$$

$$\eta \equiv \frac{\dot{\epsilon}}{H\epsilon} \simeq -\epsilon + M_{\rm Pl}^2\frac{V_{,\phi\phi}}{V} \ll 1$$



Danial Baumann (2009)

can drive inflation.

Its quantum fluctuations $\delta\phi(t,\mathbf{x}) = \phi(t,\mathbf{x}) - \langle\phi(t)\rangle$ characterized by curvature perturbations $\mathcal{R} = \frac{H}{\dot{\phi}}\delta\phi$ (in spatially flat gauge) satisfy

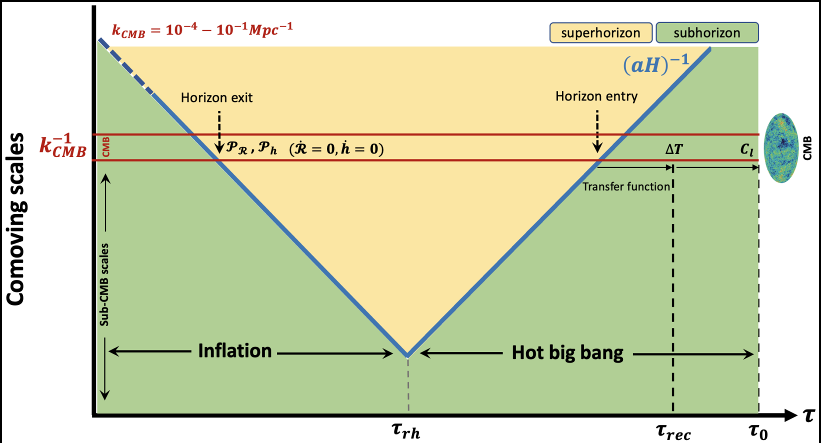
$$(a\mathcal{R}_k)'' + \left[(k\tau)^2 - 2 + \beta_{\mathcal{R}}\right]\frac{a\mathcal{R}_k}{\tau^2} = 0\,; \quad \beta_{\mathcal{R}} \equiv \frac{m_{\mathcal{R}}^2}{H_{inf}^2} \simeq 6\epsilon - 3\eta \ll 1$$

Superhorizon curvature perturbations $-k\tau < \sqrt{2}$ or $k < \sqrt{2}aH_{inf}$ are produced through interaction with gravity.

CMB observation

$$\mathcal{P}_{\mathcal{R}} = A_S \left(k/k_*\right)^{n_S - 1} \qquad \mathcal{P}_h = r\, A_S \left(k/k_*\right)^{n_T}$$

For $k_* = 0.05\text{Mpc}^{-1}$, $A_S = \mathcal{O}\left(10^{-9}\right)$, $n_S - 1 = -\mathcal{O}\left(10^{-2}\right)$, and for $k_* = 0.002\text{Mpc}^{-1}$, $r < \mathcal{O}\left(10^{-2}\right)$. $n_T = -r/8$



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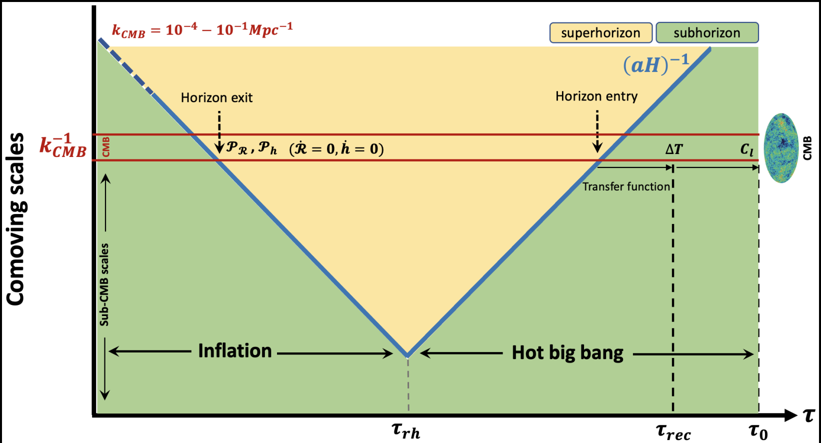
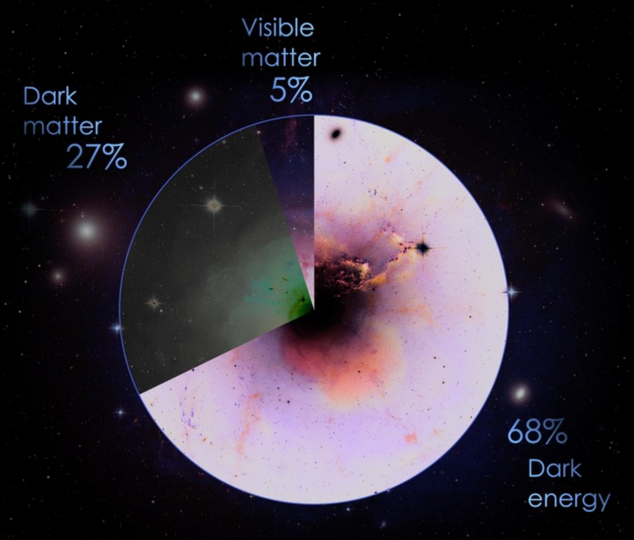
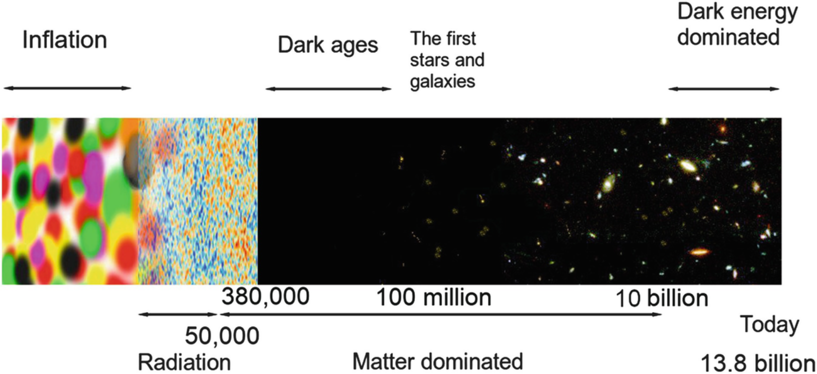


Inflation generates almost Gaussian, almost scale-invariant, and almost adiabatic perturbations

The superhorizon curvature perturbations provide the seed for the “visible matter” in the universe



https://svs.gsfc.nasa.gov/12307



Karim A. Malik & David R. Matravers, How Cosmologists Explain the Universe to Friends and Family

Can spectator fields (fields with subdominant energy) during inflation provide DM?

Outline

1. Dark matter from spectator fields during inflation
2. Spin of dark matter: Spin-0, spin-1, spin-2
3. Summary

Mass of dark matter

The dark matter spectator fields should be almost massless during inflation and massive before the time of matter and radiation equality:

- $m \ll H_{inf}$ to allow efficient particle production
- $m \gtrsim H_{M.R.}$ to make the produced particles non-relativistic before the matter-radiation equality

$$\rho_{DM} \propto \begin{cases} a^{-4} & a < a_{\rm N} \\ a^{-3} & a > a_{\rm N} \end{cases}$$

[$a_{\rm N}$ is the solution of $H(a) = m$]

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[a_{N} is the solution of $H(a) = m$]

"Roughly" speaking, we may produce dark matter particles in the mass range

$$10^{-21}\mathbf{eV} \ll m \ll 10^{15}\mathbf{GeV}$$

Spin of dark matter

The dark matter spectator fields may have different spins:

- Spin 0: Axion, $\cdots$
- Spin 1: Dark photon, $\cdots$
- Spin 2: Massive graviton, $\cdots$

Outline

Spin-0: Scalar dark matter

Massive scalar field ψ *minimally* coupled to gravity $g_{\mu\nu}$ and inflaton ϕ during inflation

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2}\mathrm{R} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) - \frac{1}{2}g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi - \frac{1}{2}m_\psi^2\psi^2$$

Since ψ is an spectator field, its contribution to the curvature perturbation is negligible. Thus, explanation of the CMB spectrum will be similar to the single field inflation.

Quantum fluctuation of ψ, $\delta\psi(t, \mathbf{x}) = \psi(t, \mathbf{x}) - \langle\psi(t)\rangle$ characterized by $\mathcal{S} \propto \delta\psi$ satisfy

$$(a\mathcal{S}_k)'' + \left[(k\tau)^2 - 2 + \beta_\mathcal{S}\right]\frac{a\mathcal{S}_k}{\tau^2} = 0\,; \qquad \beta_\mathcal{S} \equiv \frac{m_\psi^2}{H_{inf}^2}$$

Assuming $m_\psi \ll H_{inf}$, superhorizon modes $-k\tau < \sqrt{2}$ or $k < \sqrt{2}aH_{inf}$ are produced through interaction with gravity.

D. Polarski, A. A. Starobinsky, PRD (1994), T. Tenkanen, PRL (2019)

Spin-1

Massive vector field A_μ *minimally* coupled to gravity $g_{\mu\nu}$ and inflaton ϕ during inflation

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2}\mathrm{R} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m_A^2 A_\mu A^\mu$$

1 longitudinal (helicity-0): Scalar dark matter

$$A''_{k,L} + \omega_k^2 A_{k,L} = 0 \quad \omega_k^2 = \left[(k\tau)^2 - 2 + \beta_A\right]/\tau^2; \qquad \beta_A \equiv \frac{m_A^2}{H_{inf}^2}$$

It CAN be excited through gravitational particle production.

P. W. Graham, J. Mardon, S. Rajendran, PRD (2016)

2 transverse (helicity-1):

$$A''_{k,\lambda} + \omega_{k,\lambda}^2 A_{k,\lambda} = 0 \quad \omega_{k,\lambda}^2 = \left[(k\tau)^2 + \beta_A\right]/\tau^2; \qquad \lambda = (+,-)$$

It CANNOT be excited since we always have $\omega_k^2 > 0$.

Spin-1

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$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2}\mathrm{R} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m_A^2 A_\mu A^\mu$$

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$$A''_{k,\lambda} + \omega_{k,\lambda}^2 A_{k,\lambda} = 0 \quad \omega_{k,\lambda}^2 = \left[(k\tau)^2 + \beta_A\right]/\tau^2; \qquad \lambda = (+,-)$$

It CANNOT be excited since we always have $\omega_k^2 > 0$.

Is it possible to excite light $m \ll H_{inf}$ spin-1 (helicity-1) particles during inflation?

Spin-1: Conformal symmetry

The origin of this difficulty is the conformal symmetry.

The **transverse modes** can be excited through direct interactions between vector field and inflaton which breaks the conformal symmetry

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2}{\rm R} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m_A^2 A_\mu A^\mu + \mathcal{L}_{int}(A_\mu, \phi)$$

1 longitudinal (helicity-0): Scalar dark matter

$$A''_{k,L} + \omega_k^2 A_{k,L} = 0 \quad \omega_k^2 = \left[(k\tau)^2 - 2 - \alpha_{int} + \beta_A\right]/\tau^2; \qquad \beta_A \equiv \frac{m_A^2}{H_{inf}^2}$$

It CAN be excited even for $\alpha_{int} = 0$

2 transverse (helicity-1): Vector dark matter

$$A''_{k,\lambda} + \omega_{k,\lambda}^2 A_{k,\lambda} = 0 \quad \omega_{k,\lambda}^2 = \left[(k\tau)^2 - \alpha_{int} + \beta_A\right]/\tau^2; \qquad \lambda = (+,-)$$

It CAN be excited for $\alpha_{int} \neq 0$ as $\omega_k^2 < 0$ can be achieved.

Spin-1: Vector dark matter

Considering direct interactions between vector field and inflaton, some VDM models have been suggested in recent years

$$\mathcal{L} = \frac{M_{\mathrm{Pl}}^2}{2}\mathrm{R} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)$$
$$- \frac{1}{2}m_A^2 A_\mu A^\mu - \frac{f(\phi)^2}{4}F_{\mu\nu}F^{\mu\nu} - \frac{h(\phi)^2}{4}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

The mode functions $\nu_{k,\lambda} = f(\phi)A_{k,\lambda}$ satisfy

$$\nu''_{k,\lambda} + \omega^2_{k,\lambda}\nu_{k,\lambda} = 0 \qquad \omega^2_{k,\lambda} = \left[(k\tau)^2 - \alpha_{int} + \frac{\beta_A}{f^2}\right]/\tau^2$$

$$\alpha_{int} = \frac{\tau^2 f''}{f} + 2\lambda(k\tau)\frac{\tau h' h}{f^2}$$

M. Bastero-Gil, J. Santiago, L. Ubaldi, R. Vega-Morales, JCAP (2019)

K. Nakayama, JCAP (2020)

Y. Nakai, R. Namba, Z. Wang, JHEP (2020)

B. Salehian, MAG, H. Firouzjahi, S. Mukohyama, PRD (2021)

Spin-2 DM: Higuchi bound

Spin-2 field $\sigma_{\mu\nu}$ with mass m that is *minimally* coupled to gravity $g_{\mu\nu}$ and inflaton ϕ during inflation

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2}\mathrm{R} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) + \mathcal{L}_{\rm free}[\sigma_{\mu\nu}]$$

Spin-2 field has 5 dofs:

- **1 longitudinal (helicity-0)**: Scalar dark matter?
- **2 transverse (helicity-1)**: Vector dark matter?
- **2 transverse-traceless (helicity-2)**: Tensor dark matter?

For massive spin-2 particles in de Sitter space, there is a forbidden mass range (to avoid existence of ghost) [A. Higuchi, Nucl. Phys. B (1987)]

$$m^2 > 2H^2 \qquad \text{Higuchi bound}$$

As inflation is a quasi-de Sitter phase, one may expect that the deviation from the Higuchi bound will be small.

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As inflation is a quasi-de Sitter phase, one may expect that the deviation from the Higuchi bound will be small.

Is it possible to produce light $m \ll H_{inf}$ spin-2 (helicity-2) particles during inflation?

Spin-2: Tensor dark matter

The origin of the Higuchi bound is the isometries of the de Sitter space.

Similar to the case of vector field, we can **break the symmetry and relax the Higuchi bound by assuming direct interaction between the spin-2 field and inflaton**

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2}\mathrm{R} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) + \mathcal{L}_{\rm free}[\sigma_{\mu\nu}] + \mathcal{L}_{\rm int}[\sigma_{\mu\nu}, \phi]$$

In the context of EFT [L. Bordin, et al, JCAP (2018)], the mode functions for 5 helicities $\sigma_{k,I} = \left\{S_{\mathbf{k}}, V_{\mathbf{k}}^{+}, V_{\mathbf{k}}^{\times}, T_{\mathbf{k}}^{+}, T_{\mathbf{k}}^{\times}\right\}$ are [MAG, arXiv:2305.13381 [astro-ph.CO]]

$$\sigma''_{k,I} + \omega^2_{k,I}\sigma_{k,I} = 0 \qquad \omega^2_{k,I} = \left[c_I^2(k\tau)^2 - \alpha_{int} + \beta_\sigma\right]/\tau^2$$

where $c_I = \{c_s, c_v, c_v, c_t, c_t\}$ are sound speeds for the different helicities. The explicit forms of c_I, α_{int}, and the mass term $\beta_\sigma = m^2/H^2$ are determined by the EFT free parameters.

Relic density of spin-2 dark matter

Assuming instantaneous reheating

$$3M_{\rm Pl}^2 H_{\rm e}^2 = \left(\pi^2/30\right) g_{*,\rm r} T_{\rm r}^4$$

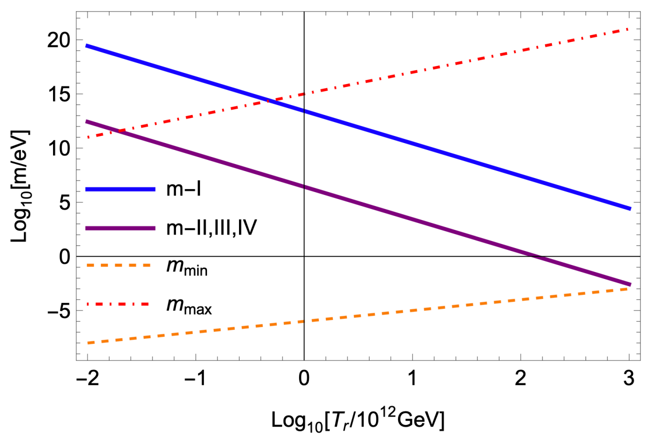
where $H_{\rm e}$ is the Hubble parameter at the end of inflation and $T_{\rm r}$ is the reheat temperature, we can estimate **the dark matter relic density today**

$$\Omega_{\rm DM,0} = \mathcal{O}\left(10^{-16}\right) \mathcal{C}_{\rm e}(c_I, \mathcal{N}_{\rm e}) \left(\frac{T_{\rm r}}{10^{12}\ \mathrm{GeV}}\right)^3 \left(\frac{m}{1\mathrm{eV}}\right)$$

The two conditions $m \ll H_{inf}$ and $m \gtrsim H_{M.R.}$ leave us with allowable mass range

$$\mathcal{O}\left(10^{-6}\right) \left(\frac{T_{\rm r}}{10^{12}\ \mathrm{GeV}}\right) \lesssim \frac{m}{1\mathrm{eV}} \ll \mathcal{O}\left(10^{15}\right) \left(\frac{T_{\rm r}}{10^{12}\ \mathrm{GeV}}\right)^2$$

Assuming the spin-2 particles provide the whole DM $\Omega_{\mathrm{DM},0} = 0.27$



Model I : $c_s, c_v, c_t = \mathcal{O}(1)$, **Model II** : $c_s, c_v = \mathcal{O}(1), c_t \ll 1$,
Model III : $c_s, c_v \ll 1, c_t = \mathcal{O}(1)$, **Model IV** : $c_s, c_v, c_t \ll 1$

$$10^{-3}\ \mathrm{eV} \lesssim m \ll 10^{5}\ \mathrm{GeV}$$

Outline

Summary

- Inflaton field generates curvature perturbation which can be probed by CMB and serve as seed for the observed large-scale structures in the Universe.
- **Spectator fields** may exist during inflation and can generate seed of **dark matter**.
- **Spin-0 spectator fields** can be generated through a very similar mechanism that inflaton generates curvature perturbation.
- **Spin-1 spectator fields** can be generated by considering direct interaction between inflaton and spectator fields (to break conformal symmetry).
- **Spin-2 spectator fields** can be generated by considering direct interaction between inflaton and spectator fields (to avoid Higuchi bound).
- While contribution of spin-0 and spin-1 spectator fields to the gravitational waves are negligible, *light* spectator spin-2 fields can give a significant contribution. [MAG and M. Sasaki, arXiv:2302.14080 [gr-qc]]
- We may be able to distinguish *light* spin-2 dark matter models from spin-0 and spin-1 cases through *gravitational wave observations*.

References

Spin 2

- MAG, *Spin-2 dark matter from inflation,*
  arXiv:2305.13381 [astro-ph.CO]

- MAG, Misao Sasaki, *Primordial-tensor-induced stochastic gravitational waves, PLB (2023),* [arXiv:2302.14080 [gr-qc]]

Spin 1

- Hassan Firouzjahi, MAG, Shinji Mukohyama, Alireza Talebian
  Dark matter from entropy perturbations in curved field space
  PRD 2022, [arXiv: 2110.09538 [gr-qc]]

- Hassan Firouzjahi, MAG, Shinji Mukohyama, Borna Salehian
  Dark photon dark matter from charged inflaton JHEP (2021),
  [arXiv:2011.06324 [hep-ph]]

Spin 0

- Borna Salehian, MAG, Hassan Firouzjahi, Shinji Mukohyama
  Vector dark matter production from inflation with symmetry breaking, PRD (2021), [arXiv:2010.04491 [hep-ph]]

MAG, *Spin-2 dark matter from inflation,* arXiv:2305.13381[astro-ph.CO]

- **Backreaction:** The energy density of the produced isocurvature modes should NOT change the background configuration of the gravity sector ($g_{\mu\nu}$) and inflaton (ϕ).

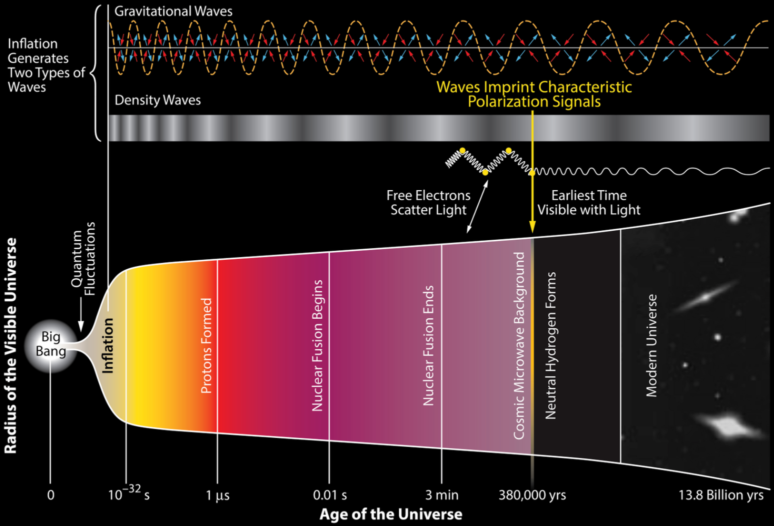
- **Isocurvature constraint:** In the case of scalar dark matter, the contribution of spectator field to the isocurvature modes is dominant and we need to take into account isocurvature bound from CMB observations at large scales.

- **Stability:** It is assumed that the spectator dark matter is decoupled from the standard model fields. Assuming some possible interactions, one finds some bounds on the corresponding couplings.

  In the case of spin-2 dark matter, there can be conversion between helicity-2 modes and gravitons. This opens interesting possibility of testing the model with gravitational waves.
  [MAG and M. Sasaki, arXiv:2302.14080 [gr-qc]]

- **Big bang nucleosynthesis:** To be more precise, $m \gg H_{M.R.}$ is not enough to make the produced dark matter modes non-relativistic.
  The dispersion relation is $\omega_l^2 = \frac{c_l^2 k^2}{a^2} + m^2$ and we need $m \gg \max[c_l k/a, H_{M.R.}]$. Some modes will be relativistic before the time of BBN and therefore contributing to $N_{\rm eff}$.

History of the Universe



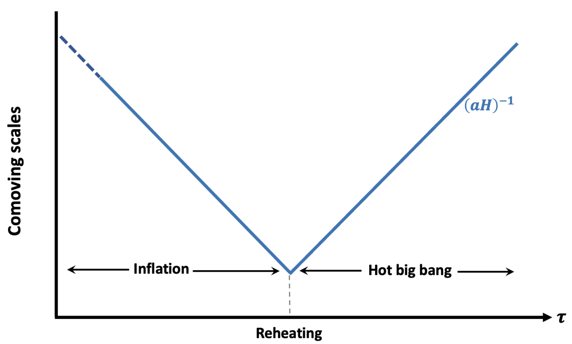
bicepkeck.org

Inflation

Inflation is a short accelerated expansion $\ddot{a} > 0$ at early times, say before the Big Bang nucleosynthesis

The comoving Hubble horizon $(aH)^{-1} = \dot{a}^{-1}$ is

- decreasing in accelerating universe ↓: $(aH)^{-1} = -\tau$ with $\tau \in [-\infty, 0]$ ($\tau \equiv \int dt/a(t)$ is conformal time)
- increasing in decelerating universe ↑: $(aH)^{-1} = \tau$ with $\tau \in [0, \infty]$



Multiple field inflation

$$S = \int \mathrm{d}^4 x \sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}\mathrm{R} - \frac{1}{2} g^{\mu\nu}\gamma_{ab}\partial_\mu\phi^a\partial_\nu\phi^b - V(\phi^a)\right]$$

$$\phi^a(t, \mathbf{x}) = \phi^a(t) + \delta\phi^a(t, \mathbf{x}) \qquad a = 1, 2$$

$$\rho = \frac{1}{2}\left[\gamma_{ab}\dot\phi^a\dot\phi^b\right] + V(\phi^a) \qquad p = \frac{1}{2}\left[\gamma_{ab}\dot\phi^a\dot\phi^b\right] - V(\phi^a)$$

The background dynamics becomes the same as single field inflation by the following identification:

$$\left[\gamma_{ab}\dot\phi^a\dot\phi^b\right] \quad \Longleftrightarrow \quad \dot\phi^2$$

Multiple field inflation

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$$\phi^a(t, \mathbf{x}) = \phi^a(t) + \delta\phi^a(t, \mathbf{x}) \qquad a = 1, 2$$

$$\rho = \frac{1}{2} \left[\gamma_{ab} \dot{\phi}^a \dot{\phi}^b \right] + V(\phi^a) \qquad p = \frac{1}{2} \left[\gamma_{ab} \dot{\phi}^a \dot{\phi}^b \right] - V(\phi^a)$$

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$$\left[\gamma_{ab} \dot{\phi}^a \dot{\phi}^b \right] \quad \Longleftrightarrow \quad \dot{\phi}^2$$

Which of $\delta\phi^1$ and $\delta\phi^2$ plays the role of curvature perturbations?

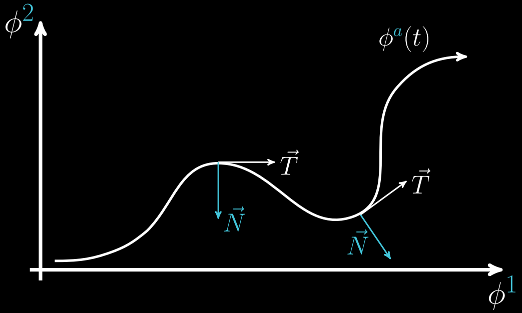
Adiabatic/entropy decomposition

$$\mathcal{R} = \frac{H}{\sqrt{\gamma_{ab}\dot{\phi}^a\dot{\phi}^b}} T_a \delta\phi^a$$

curvature perturbations

$$\mathcal{S} = \frac{H}{\sqrt{\gamma_{ab}\dot{\phi}^a\dot{\phi}^b}} N_a \delta\phi^a$$

entropy perturbations



Entropic DM

Defining $s \equiv a(\sqrt{\gamma_{ab}\dot{\phi}^a\dot{\phi}^b}/H)\mathcal{S}$ and considering geodesic trajectory in field space (T^a and N^a do not change in time)

$$s_k'' + \omega_k^2 s_k = 0 \qquad \omega_k^2 \equiv \left[(k\tau)^2 - 2 - \alpha + \beta_s\right]/\tau^2$$

$$\alpha \equiv -\epsilon M_{\rm Pl}^2 \mathbb{R} \qquad \beta_s \equiv \frac{m_s^2}{H_{inf}^2} = \frac{N^a N^b \nabla_a \nabla_b V}{H^2}$$

$\mathbb{R}$ is the Ricci scalar of the field space with metric γ_{ab}

There is an upper bound on the power spectrum of the superhorizon entropy perturbations

$$\mathcal{P}_{\mathcal{S}}/\mathcal{P}_{\mathcal{R}} \lesssim 10^{-3} \quad \Rightarrow \quad \alpha - \beta_s \lesssim -0.1$$

Spin-0: DM via scalar isocurvature modes

In two-field inflationary models, performing curvature/isocurvature decomposition, you may have

$$(a\mathcal{R}_k)'' + \left[(k\tau)^2 - 2 + \beta_{\mathcal{R}}\right]\frac{a\mathcal{R}_k}{\tau^2} = 0\,; \quad \beta_{\mathcal{R}} \equiv \frac{m_{\mathcal{R}}^2}{H_{inf}^2} \simeq \mathcal{O}(\epsilon, \eta)$$

Superhorizon curvature perturbations $-k\tau < \sqrt{2}$ excited by gravity, provide seed for the large scale structures observed in the Universe.

The isocurvature satisfies

$$(a\mathcal{S}_k)'' + \left[(k\tau)^2 - 2 + \beta_{\mathcal{S}} - \alpha\right]\frac{a\mathcal{S}_k}{\tau^2} = 0\,; \qquad \beta_{\mathcal{S}} \equiv \frac{m_s^2}{H_{inf}^2}$$

Parameter α characterizes possible interactions between the two original fields.

Dark matter relic density

The accumulated energy density of the excited entropy modes with $\omega_k^2 < 0$ at the end of inflation ($H_\mathrm{e} \sim H_{inf}$)

$$\Omega_{\mathcal{S},\mathrm{e}} = \frac{\rho_{\mathcal{S},\mathrm{e}}}{3 M_\mathrm{Pl}^2 H_\mathrm{e}^2} \qquad \Omega_{\mathcal{S},\mathrm{e}} \ll 1$$

(To avoid backreaction)

$$\rho_{\mathcal{S},\mathrm{e}} = \frac{1}{2a^2} \int_{\omega_k^2<0} \frac{d^3k}{(2\pi)^3} \left(|\mathcal{S}_k'|^2 + \left(k^2 + \frac{\beta+\alpha}{\tau^2} \right) |\mathcal{S}_k|^2 \right) \Bigg|_{\tau=\tau_\mathrm{e}}$$

The power spectrum as a function of scale can be read as

$$\Omega_\mathcal{S} = \int d \ln k \, \mathcal{P}_\mathcal{S}(k)$$