Shall Bekenstein’s area law prevail?

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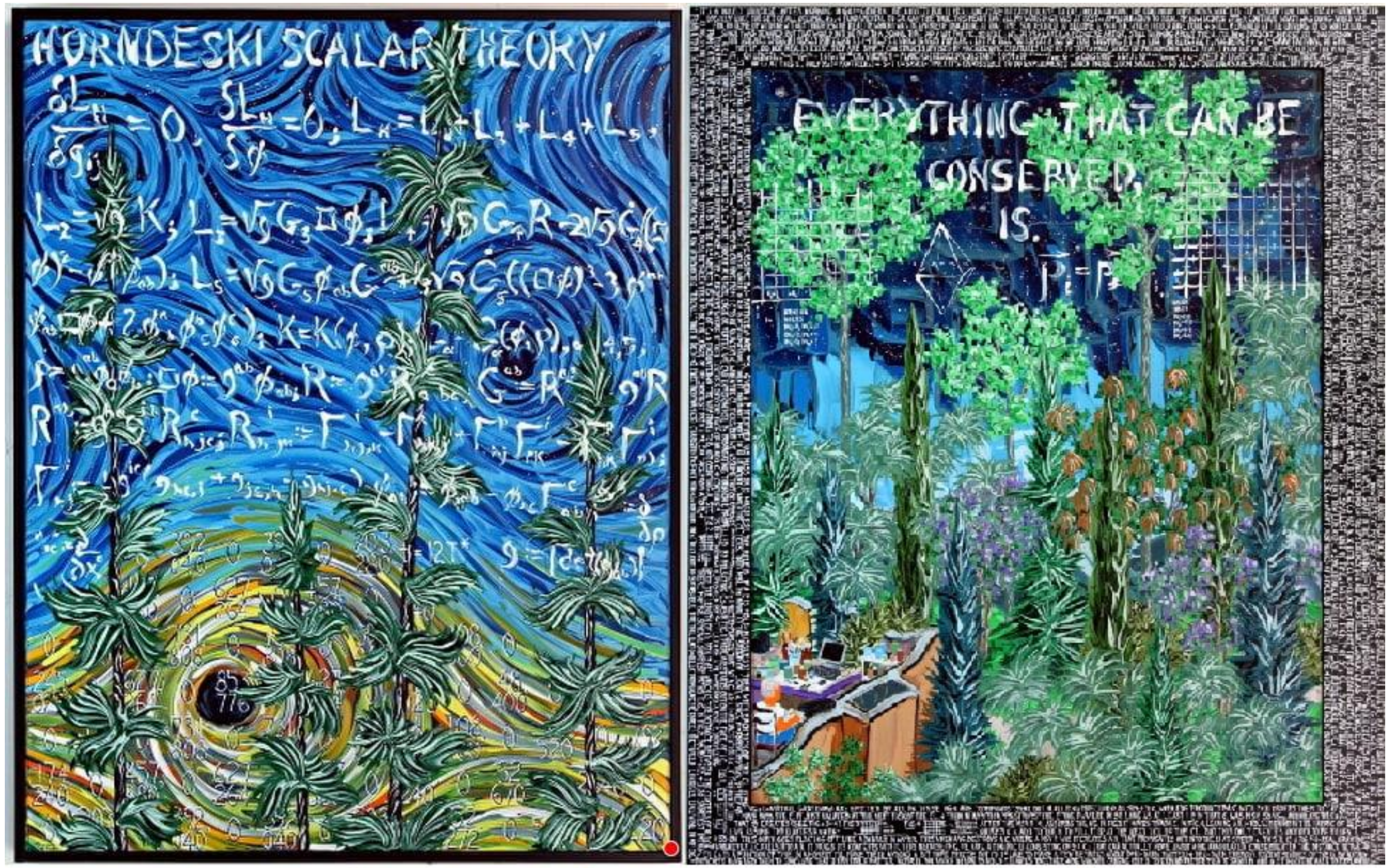

(Paintings by Gregory Horndeski)

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Scalar-tensor Einstein-Gauss-Bonnet gravity

- a Horndeski theory in 4 dimensions

$$\mathcal{L} = \frac{\sqrt{-g}}{16\pi}\left[R - 2\Lambda + \alpha\left(\phi\mathcal{G} + 4G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + 4\left(\partial\phi\right)^2\nabla_\alpha\nabla^\alpha\phi + 2\left(\partial\phi\right)^4\right)\right]$$

- $\mathcal{G}$ is the Gauss-Bonnet invariant $\mathcal{G} = R_{\alpha\beta\lambda\rho}R^{\alpha\beta\lambda\rho} - 4R_{\alpha\beta}R^{\alpha\beta} + R^2$
- in 4 dimensions $\mathcal{G} = \nabla_\mu\mathcal{G}^\mu$ ⟹ the action is invariant under a shift of ϕ by a constant $\phi \to \phi + C$ (up to a surface term)
- the theory has analytical static, spherically symmetric black hole solutions
  $\mathrm{d}s^2 = -f\left(r\right)\mathrm{d}t^2 + \mathrm{d}r^2/f\left(r\right) + r^2\mathrm{d}\Omega^2,\ f\left(r\right) = 1 + r^2/\left(2\alpha\right)\left(1 - \sqrt{1 + 8\alpha M/r^3}\right)$
  $\phi\left(r\right) = \ln\left(r/L\right) - \int_{r_+}^{r}\mathrm{d}\rho/\left(\rho\sqrt{f\left(\rho\right)}\right)$, L arbitrary (shift symmetry)

H. Lu and Y. Pang, Phys. lett. B 809 (2020) 135717;
R. A. Hennigar, D. Kubiznak, R. B. Mann and C. Pollack, JHEP 2020 (2020) 27

Black hole thermodynamics: the standard picture

- covariant phase space formalism
  - a procedure to obtain symplectic structure from the total divergence in the variation of the Lagrangian $\delta\mathcal{L} = \mathcal{E}\delta\psi + \nabla_\mu \theta^\mu[\delta]$ ← breaks shift symmetry!
  - allows to compute the Hamiltonian corresponding to Killing vector $t^\mu = (\partial/\partial t)^\mu$
  - a surface integral
  - yields the Smarr formula and the first law of black hole mechanics

V. Iyer and R. M. Wald, Phys. Rev. D 50 (1994) 846-864

Black hole thermodynamics: the standard picture

- the Smarr formula

$$M/2-(\kappa/2\pi)\left(\mathcal{A}/4+4\pi\alpha\phi\left(r_+\right)\right)+2\alpha\kappa\phi\left(r_+\right)-\alpha\left(\kappa+1/\left(2r_+\right)\right)=0$$

ADM mass

Hawking temperature

Wald entropy

potential for ϕ

potential for α

  - we have $\phi\left(r_+\right)=\ln\left(r_+/L\right)$ ⟹ $S_{\mathrm{W}}=\mathcal{A}/4+2\pi\alpha\ln\left(\mathcal{A}/L^2\right)$
  - logarithmic correction to entropy in a classical theory!

- first law of black hole thermodynamics $\delta M-T_{\mathrm{H}}\delta S_{\mathrm{W}}+2\alpha\kappa\delta\phi\left(r_+\right)=0$
- identification of temperature requires an insight from QFT ⟹ heuristic

S-W. Wei and Y.-X. Liu, Phys. Rev. D 101 (2020) 104018

Problem with the standard picture

$$M/2-(\kappa/2\pi)\left(\mathcal{A}/4+4\pi\alpha\phi\left(r_{+}\right)\right)+2\alpha\kappa\phi\left(r_{+}\right)-\alpha\left(\kappa+1/\left(2r_{+}\right)\right)=0$$

- the Smarr formula depends on the value of $\phi\left(r_{+}\right)$ (entropy and scalar charge)
- this breaks symmetry under shift of ϕ by a constant
- the equations of motion (and boundary conditions) have this symmetry
- then nothing physical in the theory should depend on the value of ϕ
- What is going on???

Role of the surface term

$$\mathcal{L} = \frac{\sqrt{-g}}{16\pi}\left[R - 2\Lambda + \alpha\left(\phi\nabla_\mu\mathcal{G}^\mu + 4G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + 4\left(\partial\phi\right)^2\nabla_\alpha\nabla^\alpha\phi + 2\left(\partial\phi\right)^4\right)\right]$$

- our action is shift symmetric only up to a surface term
- covariant phase space formalism depends on the surface terms $\delta\mathcal{L} = \mathcal{E}\delta\psi + \nabla_\mu\theta^\mu\left[\delta\right]$
- let us add a surface term $-\nabla_\mu\left(\phi\mathcal{G}^\mu\right)$ to make the shift symmetry exact

$$\mathcal{L} = \frac{\sqrt{-g}}{16\pi}\left[R - 2\Lambda + \alpha\left(-\mathcal{G}^\mu\nabla_\mu\phi + 4G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + 4\left(\partial\phi\right)^2\nabla_\alpha\nabla^\alpha\phi + 2\left(\partial\phi\right)^4\right)\right]$$

- then the covariant phase space formalism respects the shift symmetry
- the same holds for the Smarr formula and the first law
- Problem solved!

Role of the surface term

- **But wait…** surface terms are not supposed to affect the Smarr formula
  - shift the Lagrangian by some $\nabla_\mu \gamma^\mu$
  - then Wald entropy changes to $S_{\rm W} \to S_{\rm W} + \int_{\mathcal{H}} 2\gamma^{[\mu}t^{\nu]}\mathrm{d}\Sigma_{\mu\nu}$ ← integral over a horizon spatial cross-section
  - any term proportional to Killing vector t^μ integrated over $\mathcal{H}$ vanishes

  ⟹ no change in entropy

- **Resolution:** the Gauss-Bonnet $-\nabla_\mu\left(\phi\mathcal{G}^\mu\right)$ term is special
- for spacetimes with a Killing vector we have $\mathcal{G}^\mu = -2\left(\partial\mathcal{G}/\partial R_{0\mu\nu\rho}\right)\nabla_\nu t_\rho$ ← depends on a derivative of t^μ
- compare with Wald entropy $S_{\rm W} = (2\pi/\kappa)\int_{\mathcal{H}}\left(\partial\mathcal{L}/\partial R_{\mu\nu\rho\sigma}\right)\nabla_\rho t_\sigma \mathrm{d}\Sigma_{\mu\nu}$
- same structure ⟹ surface term contributes to entropy!

A. Yale, T. Padmanabhan, Gen. Rel. Grav. 43 (2011) 1549-1570;
B. S. Sarkar, A. C. Wall, Phys. Rev. D 83 (2011) 124048

Black hole thermodynamics: our proposal

- the Smarr formula

$$M/2 - (\kappa/2\pi)\left(1 + 2\alpha/r_+^2\right)\mathcal{A}/4 - \alpha/\left(2r_+\right) = 0$$

- first law of black hole thermodynamics

$$\delta M - (\kappa/2\pi)\left(1 + 2\alpha/r_+^2\right)\delta\mathcal{A}/4 = 0$$

δS_{W}

- we have $S_{\mathrm{W}} = \mathcal{A}/4$ ⟹ Bekenstein entropy of GR!

D. Kubizňák, ML, arXiv:2307.16201
ML, D. Kubizňák, R. A. Hennigar, arXiv:2309.05629

Modified temperature?

$$\delta M - (\kappa/2\pi)\left(1 + 2\alpha/r_+^2\right)\delta\mathcal{A}/4 = 0$$

- the Hawking temperature appears to be $T_{\mathrm{H}} = (\kappa/2\pi)\left(1 + \alpha/\left(2r_+^2\right)\right)$
- but the Hawking effect is kinematic, how can the temperature be modified

  A: propagation speed of gravitons in scalar-tensor theories does not equal c
  the Hawking effect can thus be modified

  B: there is a "screening mechanism" affecting propagation of the emitted particles
- Brown-York construction of the Euclidean grand canonical ensemble
  - allows to compute temperature independently of entropy
  - for the shift symmetric action, it reproduces the modified temperature!

K. Hajian, S. Liberati, M. M. Sheikh-Jabbari, M. H. Vahidinia, Phys. Lett. B 812 (2020) 136002;
H. W. Braden, J. D. Brown, B. F. Whiting, and York, Phys. Rev. D 42, 3376 (1990)

Summary

- the “standard” way to do black hole thermodynamics cannot be blindly trusted
- scalar-tensor Einstein-Gauss-Bonnet gravity illustrates this:

1) Gauss-Bonnet term in 4D is a surface term that affects entropy
2) the Hawking effect appears to be modified

- What happens in JT/Liouville gravity?
- What about Lovelock theories?

Thank you!