Effective field theory of black hole perturbations with timelike scalar profile

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Ref. arXiv: 2204.00228 w/ V.Yingcharoenrat
arXiv: 2208.02943 w/ K.Takahashi & V.Yingcharoenrat
arXiv: 2304.14304 w/ K.Takahashi & K.Tomikawa & V.Yingcharoenrat

Also Arkani-Hamed, Cheng, Luty and Mukohyama 2004 (hep-th/0312099)
Mukohyama 2005 (hep-th/0502189)

Collaborators

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K.Takahashi

K.Tomikawa

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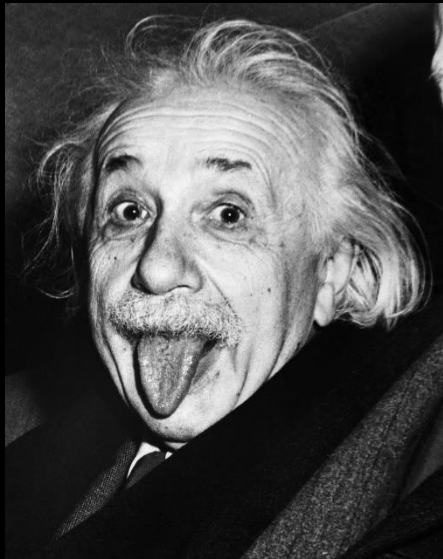
Why gravity beyond GR?

(GR : general relativity)

- Challenging **mysteries in the universe**
  Dark energy, dark matter, inflation, big-bang singularity, cosmic magnetic field and tensions
- Necessary for **quantum gravity**
  Superstring, Horava-Lifshitz, LQG, etc.
- **Testing GR**
  One of the best ways to test GR is to make predictions and to compare them with observations/experiments.
- …



Some examples (my personal experiences)

I. Effective field theory (EFT) approach
   IR modification of gravity
   motivation: dark energy/inflation, universality
II. Massive gravity
   IR modification of gravity
   motivation: “Can graviton have mass?” & dark energy
III. Minimally modified gravity
   IR modification of gravity
   motivation: tensions in cosmology, various constraints
IV. Horava-Lifshitz gravity
   UV modification of gravity
   motivation: quantum gravity
V. Superstring theory
   UV modification of gravity
   motivation: quantum gravity, unified theory

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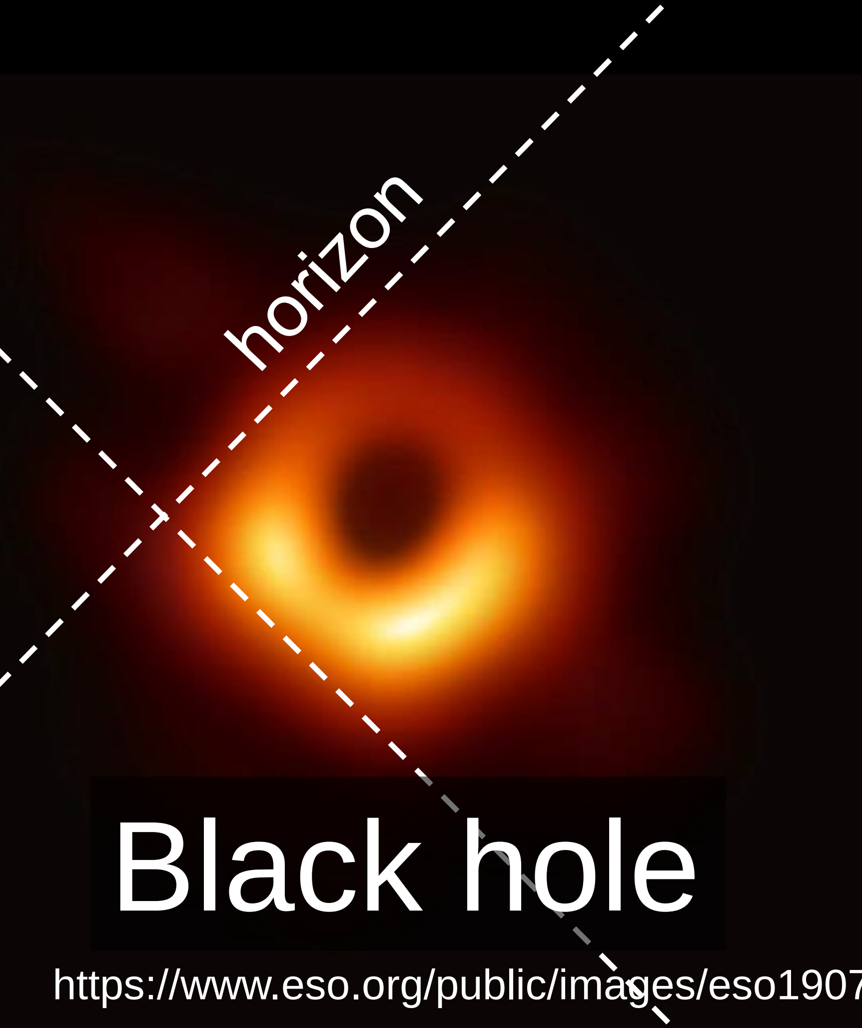
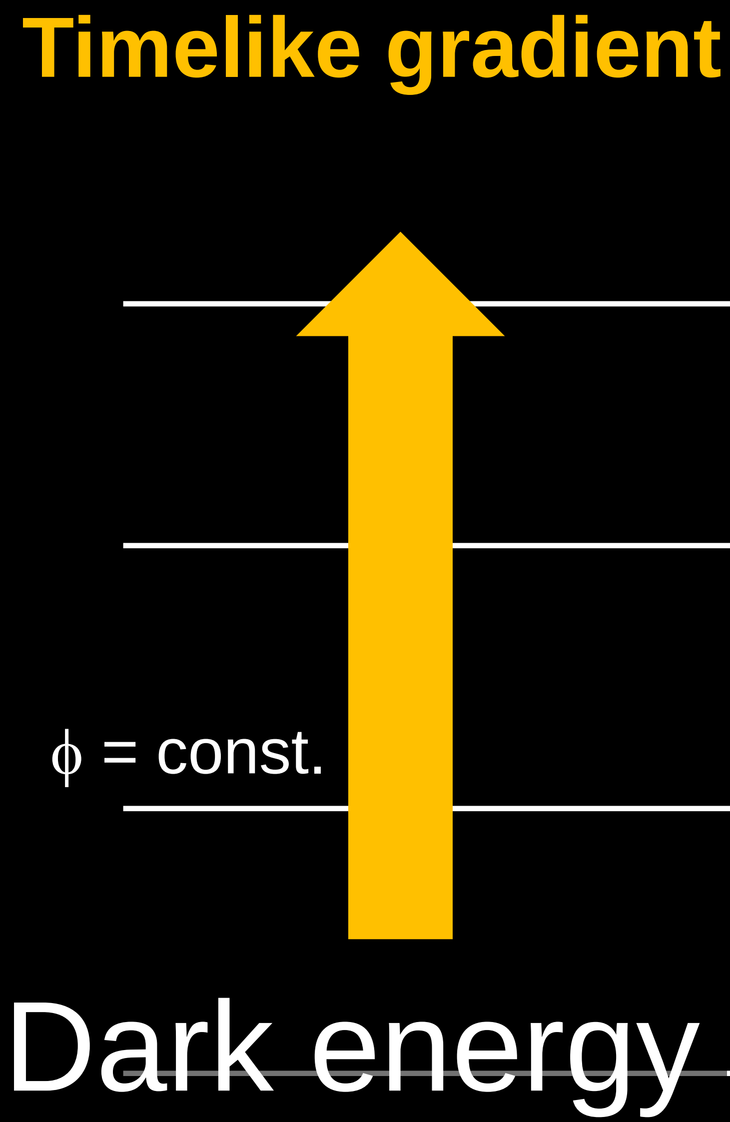
Proto-type of modified gravity: scalar-tensor theory

- Metric $g_{\mu\nu}$ + scalar field ϕ
- Jordan (1955), Brans & Dicke (1961), Bergmann (1968), Wagoner (1970), …
- Most general scalar-tensor theory of gravity with 2^{nd} order covariant EOM: Horndeski (1974)
- DHOST theories beyond Horndeski: Langlois & Noui (2016)
- U-DHOST theories beyond DHOST: DeFelice, Langlois, Mukohyama, Noui & Wang (2018)
- All of them (and more) are universally described by an effective field theory (EFT)

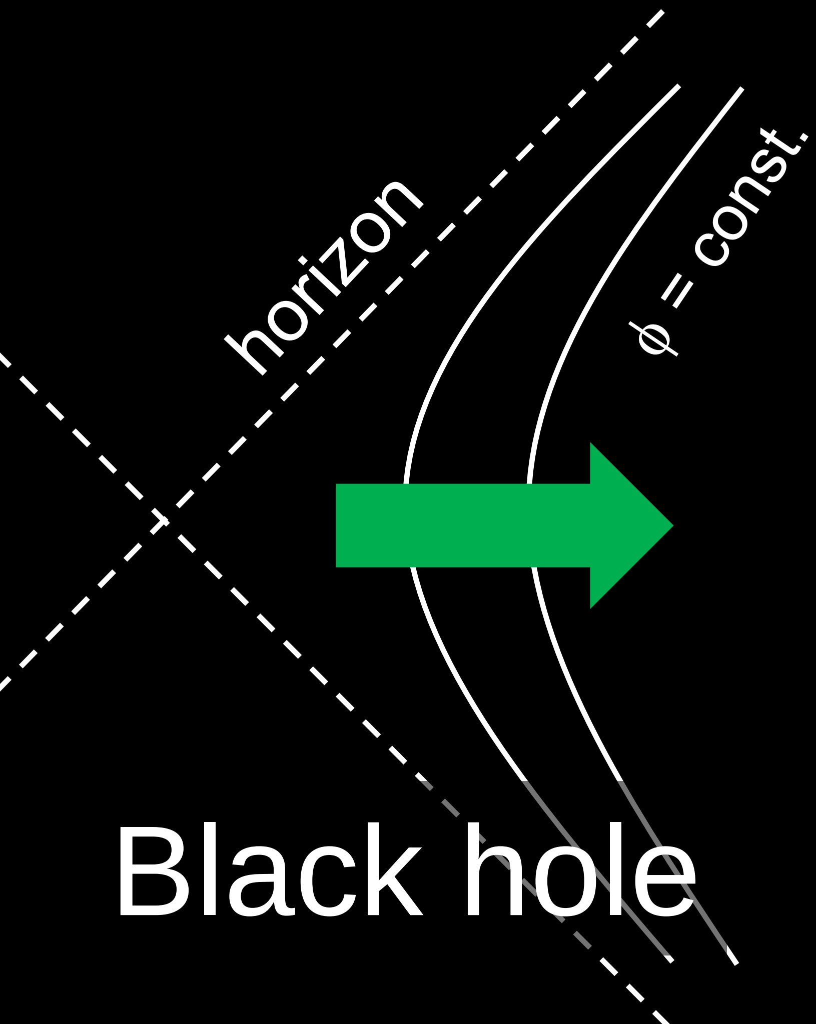
- Cosmology and black holes (BHs) play as important roles in gravitational physics as blackbody radiation and hydrogen atoms did in quantum mechanics.
- In cosmology a time-dependent scalar field can act as dark energy (DE), while BHs serve as probes of strong gravity. We then hope to learn something about the EFT of DE by BHs.


horizon
Black hole
https://www.eso.org/public/images/eso1907a/


Timelike gradient
ϕ = const.
Dark energy

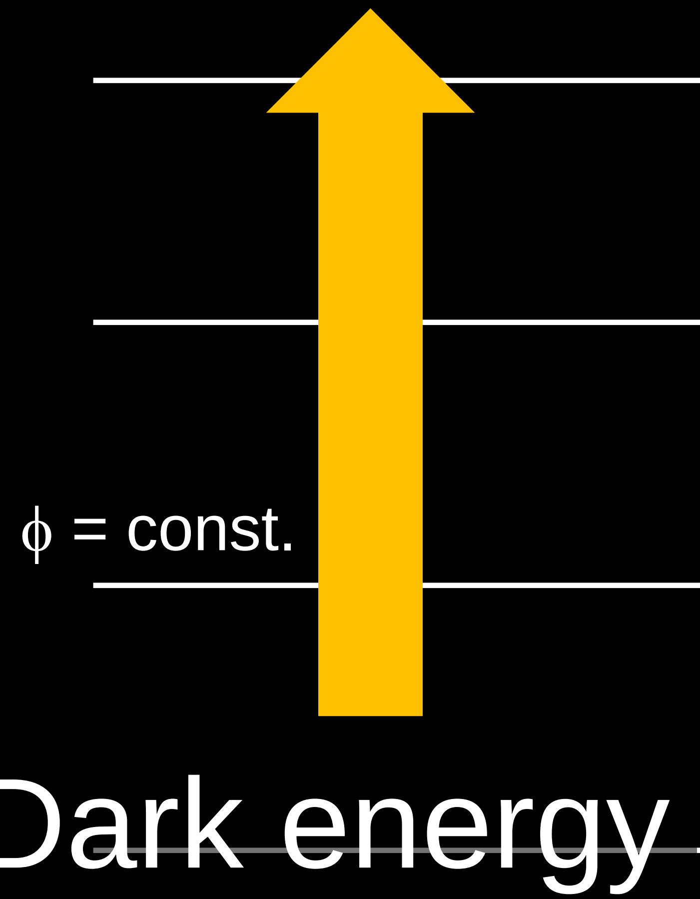
Unlucky case

Spacelike gradient

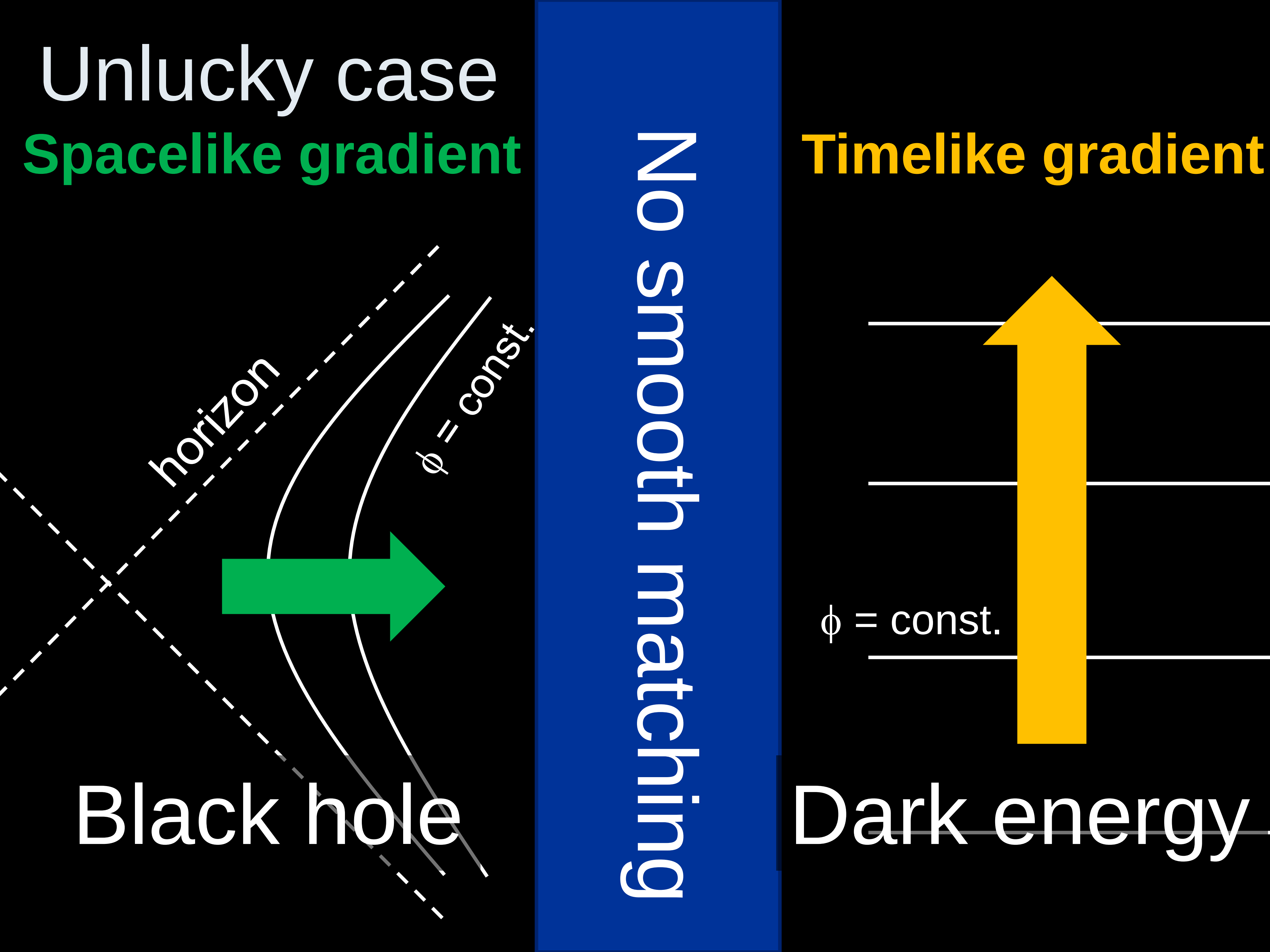


Black hole

Timelike gradient

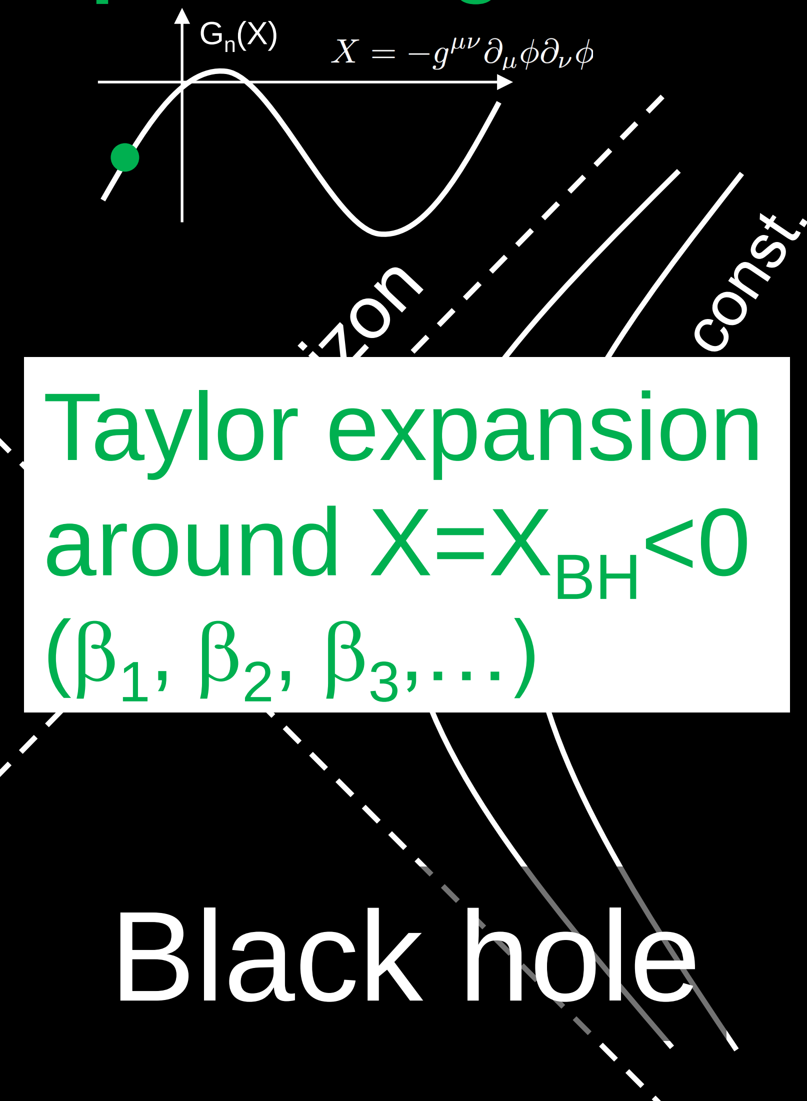


Dark energy


Unlucky case
Spacelike gradient
horizon
ϕ = const.
Black hole
No smooth matching
Timelike gradient
ϕ = const.
Dark energy

Unlucky case

Spacelike gradient

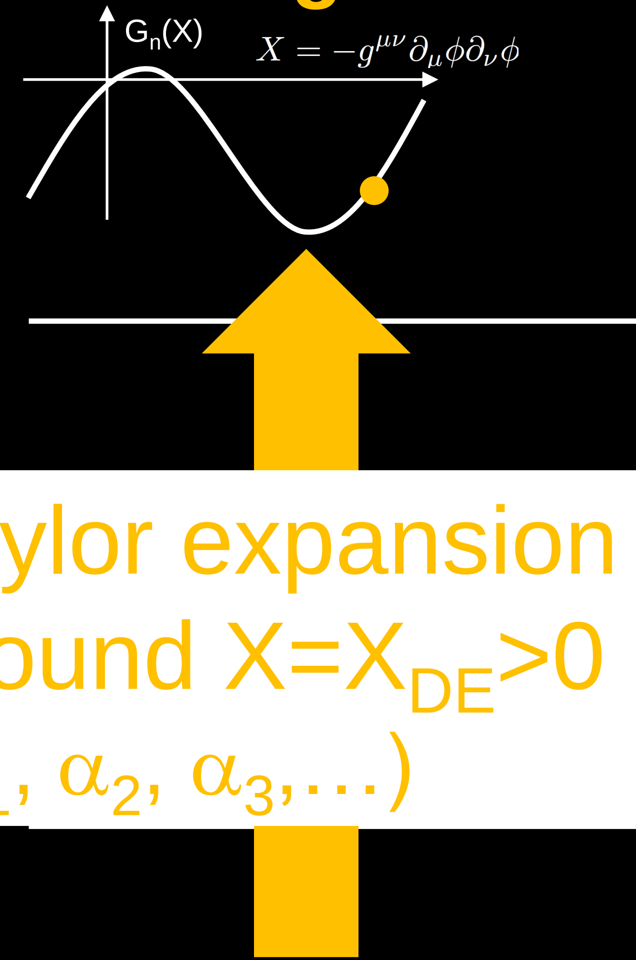


Taylor expansion around $X=X_{BH}<0$ $(\beta_1, \beta_2, \beta_3, \ldots)$

Black hole

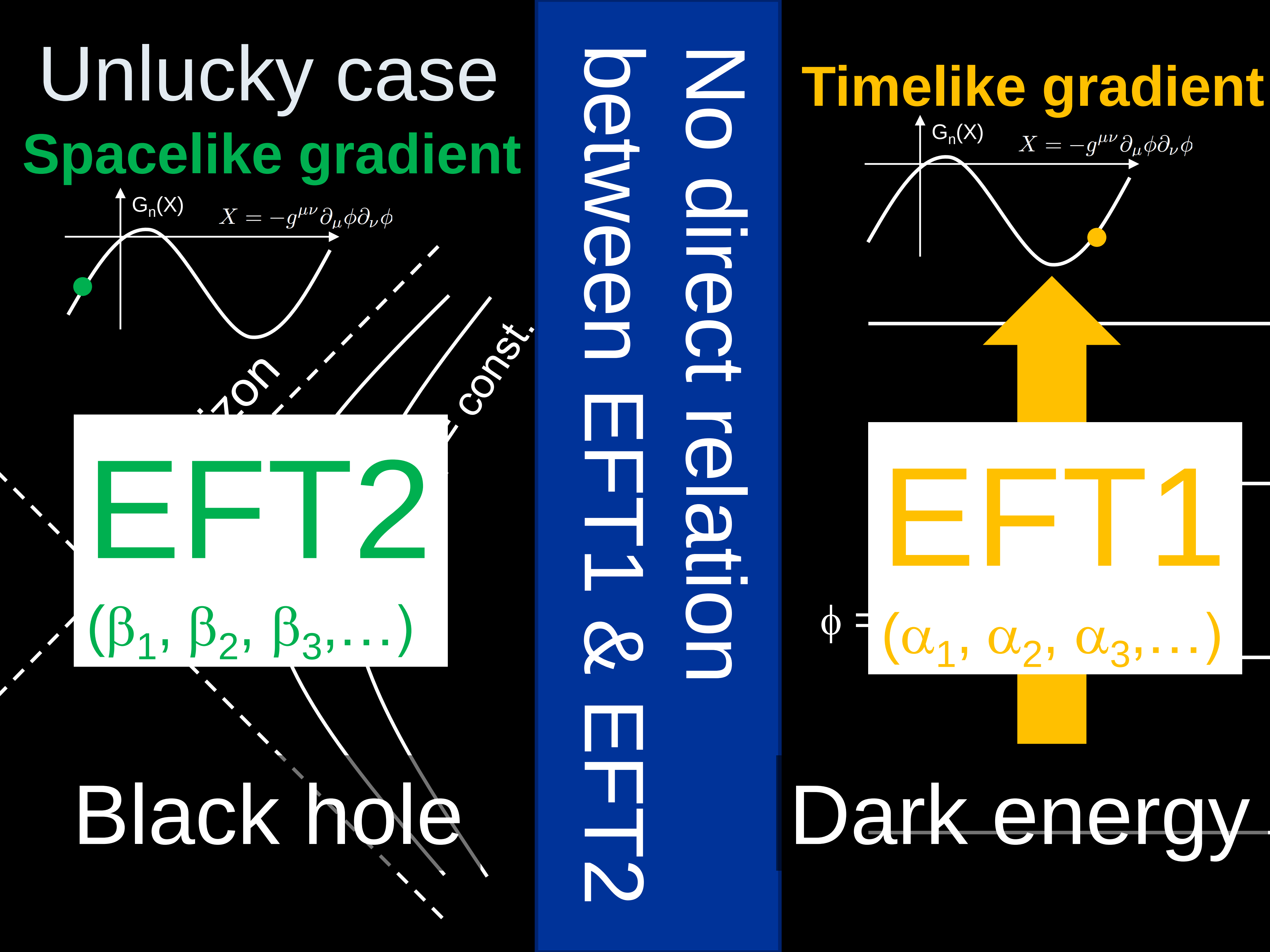
No direct relation between Taylor coefficients

Timelike gradient



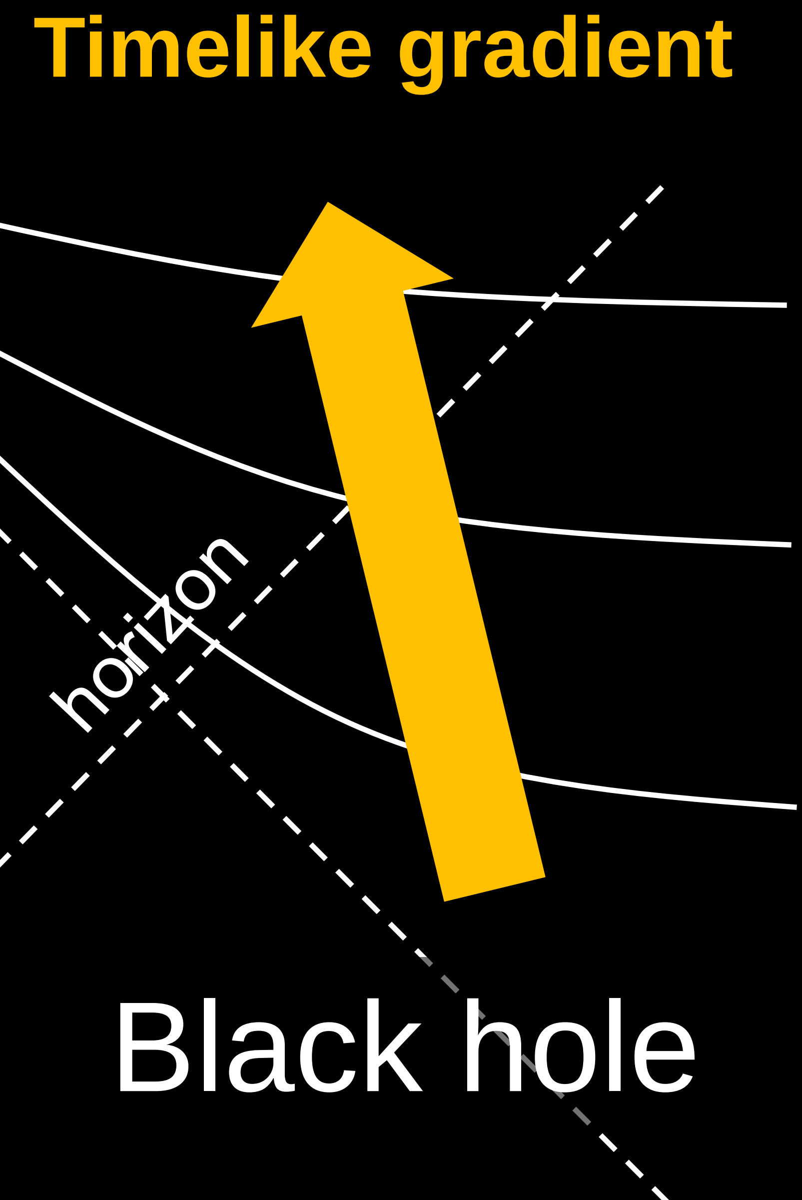
Taylor expansion around $X=X_{DE}>0$ $(\alpha_1, \alpha_2, \alpha_3, \ldots)$

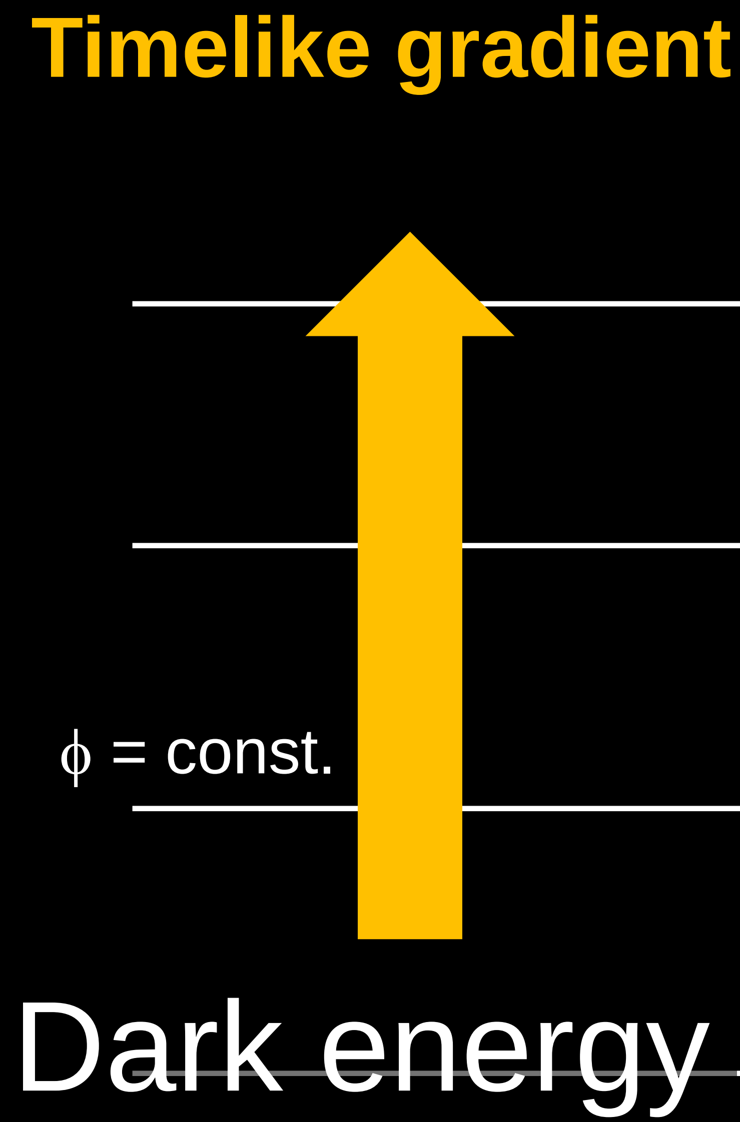
Dark energy



Unlucky case
Spacelike gradient
$G_n(X)$
$X = -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$
const.
EFT2
$(\beta_1, \beta_2, \beta_3, \ldots)$
Black hole
No direct relation between EFT1 & EFT2
Timelike gradient
$G_n(X)$
$X = -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$
EFT1
$(\alpha_1, \alpha_2, \alpha_3, \ldots)$
Dark energy

Lucky case


Timelike gradient
horizon
Black hole


Timelike gradient
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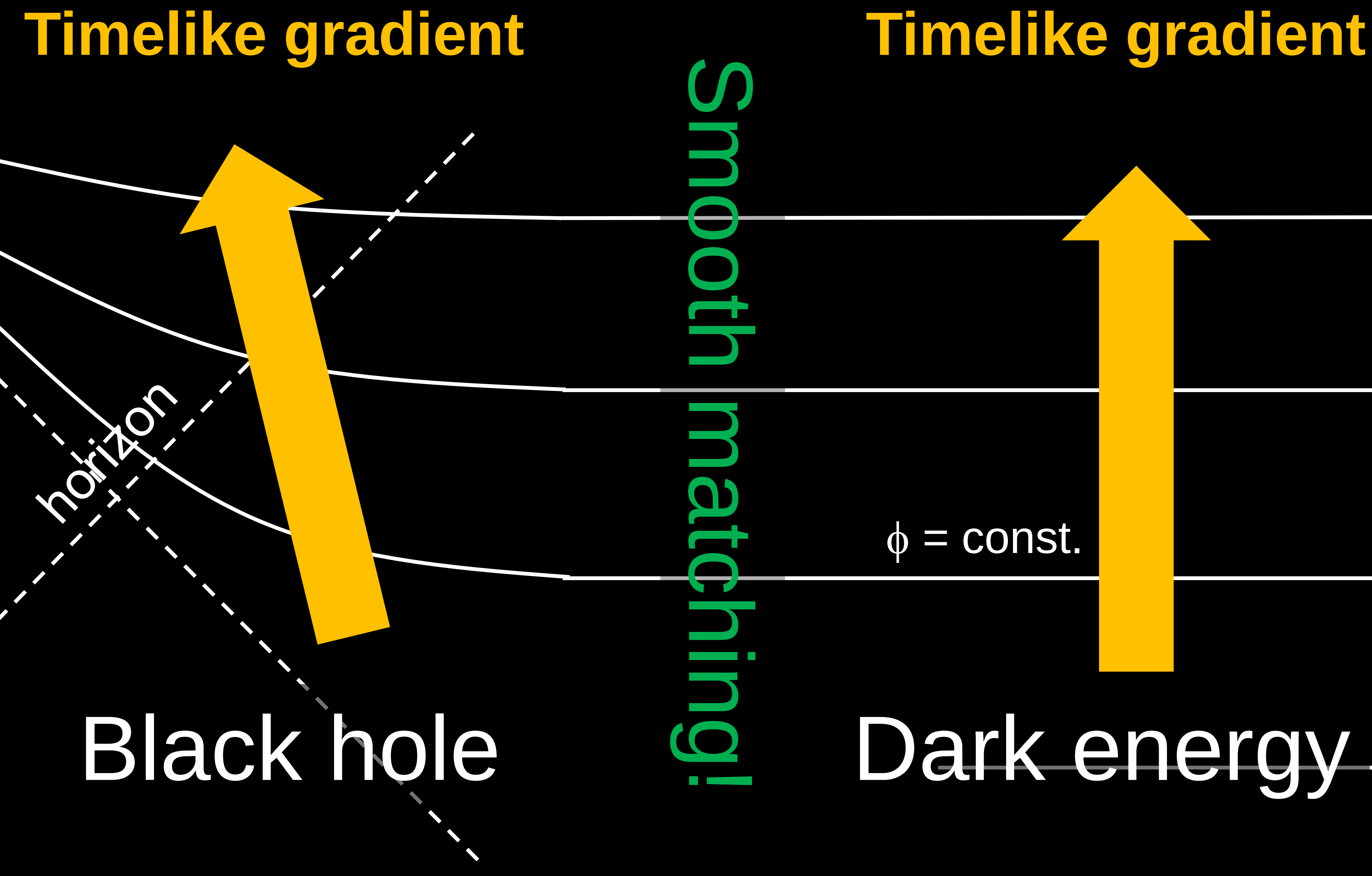
Dark energy

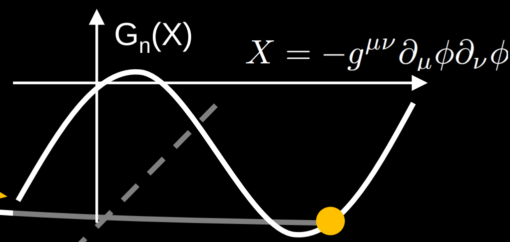
Lucky case



Timelike gradient
Timelike gradient
Smooth matching!
horizon
φ = const.
Black hole
Dark energy

Lucky case

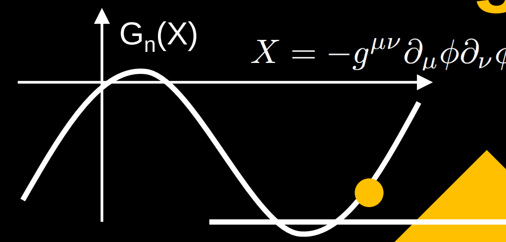
Timelike gradient



Taylor expansion around $X=X_{BH}>0$
(α'_1, α'_2, α'_3,…)

Black hole

Timelike gradient



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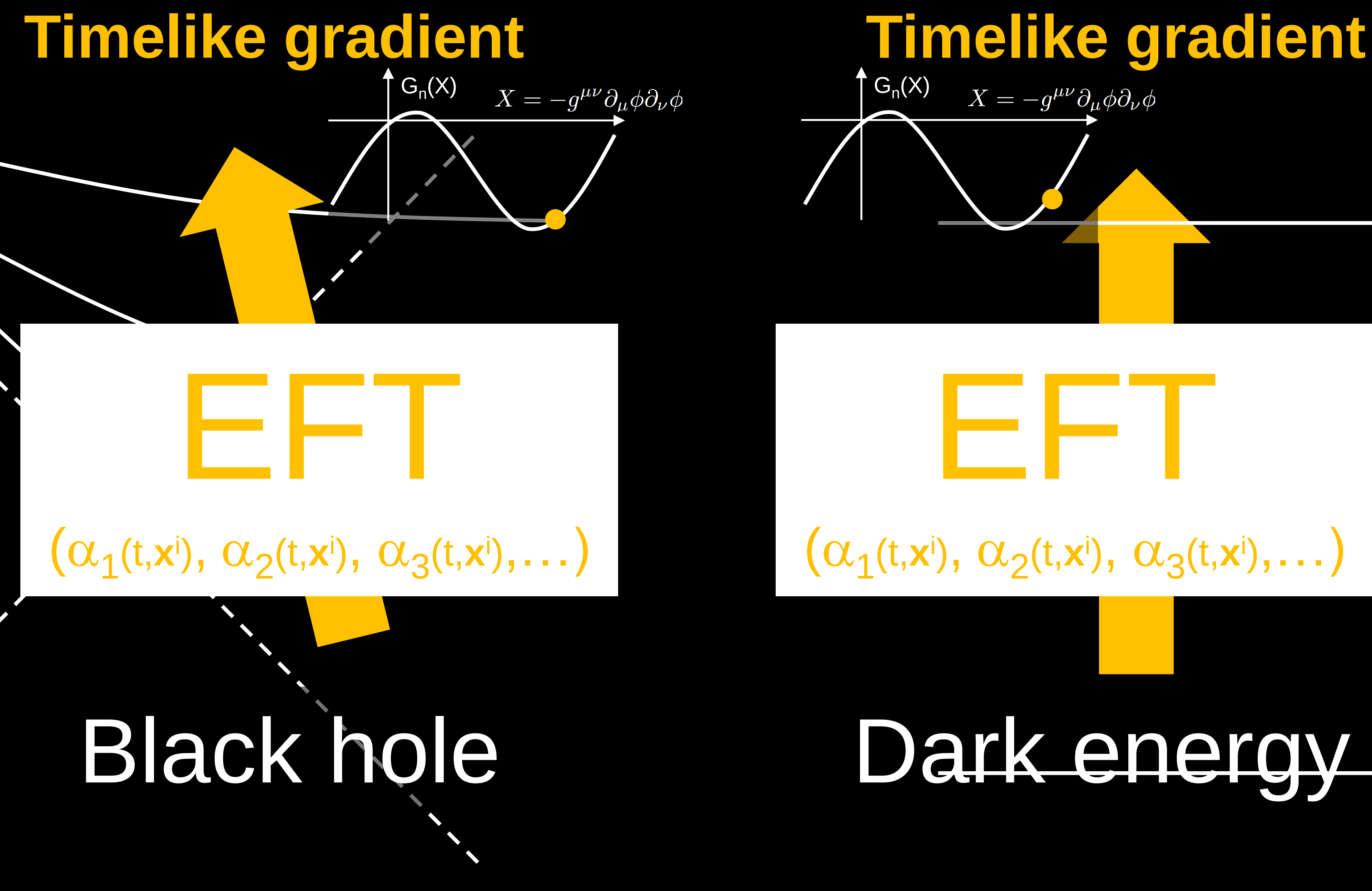
Dark energy

Lucky case


Timelike gradient
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$G_n(X)$
$X = -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$
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EFT
(α_1(t,**x**^{i}), α_2(t,**x**^{i}), α_3(t,**x**^{i}),...)
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- In cosmology a time-dependent scalar field can act as dark energy (DE), while BHs serve as probes of strong gravity. We then hope to learn something about the EFT of DE by BHs.
- This would require **the scalar field profile to be timelike near BH**. Otherwise, the two EFTs, one for DE and the other for BH, can be unrelated to each other (unless a UV completion is specified).

Stealth solutions in k-essence

Mukohyama 2005

- Action in Einstein frame

$$I = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R + P(X)\right] \qquad X = -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

- EOMs

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}P'(X)g^{\mu\nu}\partial_\nu\phi\right) = 0$$

$$M_{\rm Pl}^2 G_{\mu\nu} = 2P'(X)\partial_\mu\phi\partial_\nu\phi + P(X)g_{\mu\nu}$$

- Stealth sol with $X = X_0$, where $P'(X_0)=0$

$$G_{\mu\nu} = \Lambda_{\rm eff}\, g_{\mu\nu} \qquad \Lambda_{\rm eff} = P(X_0)/M_{\rm Pl}^2$$

- $X = X_0$ ($\neq 0$)

⟺ $u^\mu = g^{\mu\nu}\partial_\nu\phi$ defines geodesic congruence

$$(u^\nu\nabla_\nu u^\mu = -\nabla^\mu X/2 = 0)$$

⟺ $\phi/\sqrt{|X_0|}$ **defines Gaussian normal coord.**

Stealth solutions with $\phi = qt+\psi(r)$

- Schwarzschild in k-essence (Mukohyama 2005)
- Schwarzschild-dS in Horndeski theory (Babichev & Charmousis 2013, Kobayashi & Tanahashi 2014) Schwarzshild-dS in DHOST (Ben Achour & Liu 2019, Motohashi & Minamitsuji 2019)
- Kerr-dS in DHOST (Charmousis & Crisotomi & Gregory & Stergioulas 2019)
- However, perturbations around most of those stealth solutions are infinitely strongly coupled (de Rham & Zhang 2019) . This means the solutions cannot be trusted.
- Fortunately, Scordatura (= detuning of degeneracy condition) solves the strong coupling problem (Motohashi & Mukohyama 2019), if and only if the scalar profile is timelike.
- EFT of ghost condensation already includes scordatura (Arkani-Hamed & Cheng & Luty & Mukohyama 2004)
- Approximate Schwarzschild in ghost condensation (Mukohyama 2005). Also in quadratic HOST (DeFelice & Mukohyama & Takahashi, JCAP 03 (2023) 050).

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EFT of scalar-tensor gravity with timelike scalar profile

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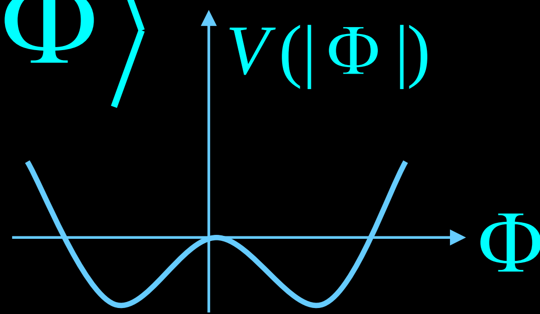
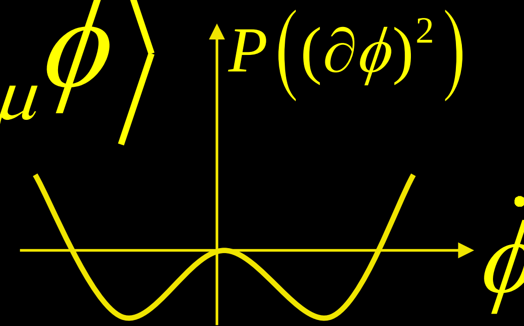
- **Time diffeo is broken by the scalar profile but spatial diffeo is preserved.**
- All terms that respect spatial diffeo must be included in the EFT action.
- Derivative & perturbative expansions
- Diffeo can be restored by introducing NG boson

EFT on Minkowski background

= ghost condensation

Arkani-Hamed, Cheng, Luty and Mukohyama, JHEP 0405:074,2004

	Higgs mechanism	***Ghost condensate*** Arkani-Hamed, Cheng, Luty and Mukohyama 2004
Order parameter	$\langle \Phi \rangle$ 	$\langle \partial_\mu \phi \rangle$ 
Instability	Tachyon $-\mu^2\Phi^2$	Ghost $-\dot{\phi}^2$
Condensate	V’=0, V”>0	P’=0, P”>0
Broken symmetry	Gauge symmetry	Time diffeomorphism
Force to be modified	Gauge force	Gravity
New force law	Yukawa type	Newton+Oscillation

EFT of ghost condensation =
EFT of scalar-tensor gravity with timelike scalar profile on Minkowski background

Arkani-Hamed, Cheng, Luty and Mukohyama 2004

Backgrounds characterized by

- ✧ $\langle \partial_\mu \phi \rangle = const \neq 0$ and timelike
- ✧ Minkowski metric

t → t + const & t → -t unbroken

up to ϕ → ϕ + const & ϕ → -ϕ

$$\Rightarrow \quad L_{eff} = L_{EH} + M^4 \left\{ \left(h_{00} - 2\dot{\pi} \right)^2 - \frac{\alpha_1}{M^2} \left(K + \vec{\nabla}^2 \pi \right)^2 - \frac{\alpha_2}{M^2} \left(K^{ij} + \vec{\nabla}^i \vec{\nabla}^j \pi \right) \left(K_{ij} + \vec{\nabla}_i \vec{\nabla}_j \pi \right) + \cdots \right\}$$

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EFT on Minkowski background = ghost condensation

Arkani-Hamed, Cheng, Luty and Mukohyama, JHEP 0405:074,2004

EFT on cosmological background = EFT of inflation/dark energy

Creminelli, Luty, Nicolis and Senatore 2006
Cheung, Creminelli, Fitzpatrick, Kaplan and Senatore 2007

Application: non-Gaussinity of inflationary perturbation $\zeta = -H\pi$

$$I_\pi = M_{Pl}^2 \int dt d^3\vec{x}\, a^3 \left\{ -\frac{\dot{H}}{c_s^2}\left(\dot{\pi}^2 - c_s^2 \frac{(\partial_i \pi)^2}{a^2}\right) \right.$$

$$\left. -\dot{H}\left(\frac{1}{c_s^2} - 1\right)\left(\frac{c_3}{c_s^2}\dot{\pi}^3 - \dot{\pi}\frac{(\partial_i\pi)^2}{a^2}\right) + O(\pi^4, \tilde{\epsilon}^2) + L^{(2)}_{\tilde{\delta}K, \tilde{\delta}R} \right\}$$

2 types of 3-point interactions

➡ power spectrum

$$P_\zeta(\vec{k}) = \frac{\Delta}{k^3}, \quad \Delta = \left.\frac{H^4}{-4M_{Pl}^2 \dot{H} c_s}\right|_{c_s k \simeq aH}$$

➡ non-Gaussianity

$$\langle \zeta_{\vec{k}_1}(t)\, \zeta_{\vec{k}_2}(t)\, \zeta_{\vec{k}_3}(t)\rangle = (2\pi)^3 \delta^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) B_\zeta$$

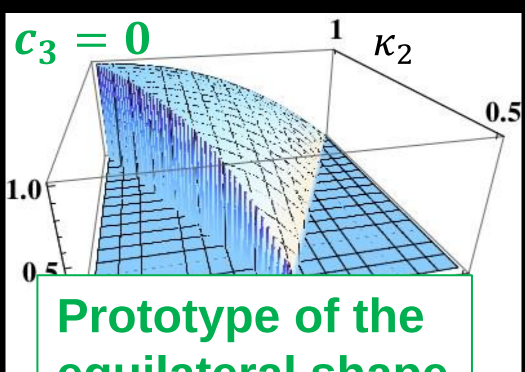
c_s^2 → size of non-Gaussianity

$$k^6 B_\zeta|_{k_1=k_2=k_3=k} = \frac{18}{5}\Delta^2\left(f_{NL}^{\dot{\pi}(\partial_i\pi)^2} + f_{NL}^{\dot{\pi}^3}\right)$$

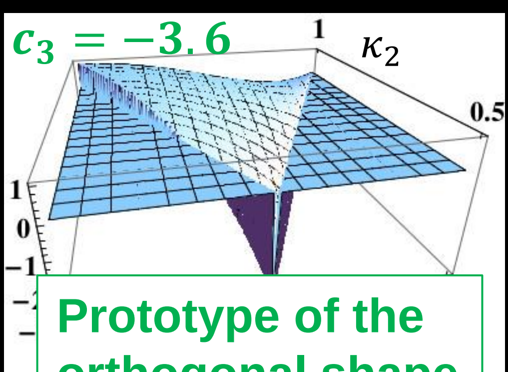
$$f_{NL}^{\dot{\pi}(\partial_i\pi)^2} = \frac{85}{324}\left(1 - \frac{1}{c_s^2}\right) \qquad f_{NL}^{\dot{\pi}^3} = \frac{5 c_3}{81}\left(1 - \frac{1}{c_s^2}\right) \qquad \propto \frac{1}{c_s^2} \text{ for small } c_s^2$$

c_3 → shape of non-Gaussianity

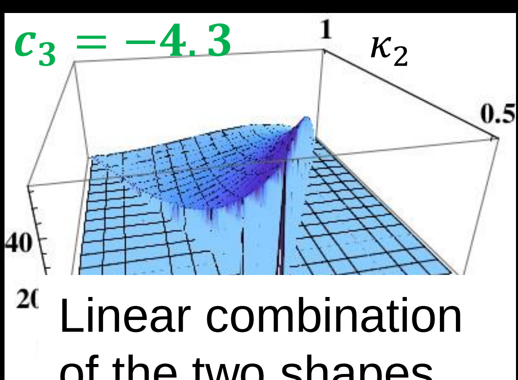
plots of $B_\zeta(k, \kappa_2 k, \kappa_3 k)/B_\zeta(k,k,k)$



Prototype of the equilateral shape



Prototype of the orthogonal shape



Linear combination of the two shapes

Parametrization suitable for DE → EFT of DE

Gubitosi, Piazza, Vernizzi 2012
Gleyzes, Langlois, Piazza, Vernizzi 2013

- Matter (in addition to DE) needs to be added → Jordan frame description is convenient
- In Jordan frame the coefficient of the 4d Ricci scalar is not constant.

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\,\Big[\boxed{M_*^2 f R} - \rho_D + p_D - M_*^2(5H\dot{f} + \ddot{f}) - \Big(\rho_D + p_D + M_*^2(H\dot{f} - \ddot{f})\Big)\,g^{00}$$
$$+ M_2^4(\delta g^{00})^2 - \bar{m}_1^3\,\delta g^{00}\delta K - \bar{M}_2^2\,\delta K^2 - \bar{M}_3^2\,\delta K_\mu{}^\nu\delta K^\mu{}_\nu + m_2^2 h^{\mu\nu}\partial_\mu g^{00}\partial_\nu g^{00}$$
$$+ \lambda_1\delta R^2 + \lambda_2\delta R_{\mu\nu}\delta R^{\mu\nu} + \mu_1^2\delta g^{00}\delta R + \gamma_1 C^{\mu\nu\rho\sigma}C_{\mu\nu\rho\sigma} + \gamma_2\epsilon^{\mu\nu\rho\sigma}C_{\mu\nu}{}^{\kappa\lambda}C_{\rho\sigma\kappa\lambda}$$
$$+ \frac{M_3^4}{3}(\delta g^{00})^3 - \bar{m}_2^3\,(\delta g^{00})^2\delta K + \dots\Big]\,,$$

EFT of scalar-tensor gravity with timelike scalar profile

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EFT on cosmological background	= EFT of inflation/dark energy Creminelli, Luty, Nicolis and Senatore 2006 Cheung, Creminelli, Fitzpatrick, Kaplan and Senatore 2007
EFT on arbitrary background	= EFT of BH perturbations Mukohyama and Yingcharoenrat, JCAP 09 (2022) 010

Taylor expansion of the general action $S = \int d^4x\sqrt{-g}\; F(R_{\mu\nu\alpha\beta}, g^{\tau\tau}, K_{\mu\nu}, \nabla_\nu, \tau)$

$$S = \int d^4x\sqrt{-g}\left[\bar{F} + \bar{F}_{g^{\tau\tau}}\delta g^{\tau\tau} + \bar{F}_K \delta K + \quad \dots\right]$$

Consistency relations ⟸ S is invariant under spatial diffeo but the background breaks it.

$$\frac{d}{dx^i}\bar{F} = \bar{F}_{g^{\tau\tau}}\frac{\partial \bar{g}^{\tau\tau}}{\partial x^i} + \bar{F}_K\frac{\partial \bar{K}}{\partial x^i} + \dots$$

Applications to BHs with timelike scalar profile

- Background analysis for spherical BH
  [arXiv: 2204.00228 w/ V.Yingcharoenrat]

Background analysis

- Spherically symmetric, static background

$$ds^2 = -A(r)dt^2 + \frac{dr^2}{B(r)} + r^2 d\Omega^2$$

- Lemaitre coordinates

$$ds^2 = -d\tau^2 + [1 - A(r)]d\rho^2 + r^2 d\Omega^2$$

- Shift and Z_2 symmetries

$$\Phi \to \Phi + const. \qquad \Phi \to -\Phi$$

$$S = \int d^4x\sqrt{-g}\Big[\frac{M_\star^2}{2}R - \Lambda(r) - c(r)g^{\tau\tau} - \tilde{\beta}(r)K - \alpha(r)\bar{K}^\mu_\nu K^\nu_\mu - \zeta(r)n^\mu\partial_\mu g^{\tau\tau}$$
$$+ \frac{1}{2}m_2^4(r)(\delta g^{\tau\tau})^2 + \frac{1}{2}\tilde{M}_1^3(r)\delta g^{\tau\tau}\delta K + \frac{1}{2}M_2^2(r)\delta K^2 + \frac{1}{2}M_3^2(r)\delta K^\mu_\nu\delta K^\nu_\mu$$
$$+ \frac{1}{2}\mu_1^2(r)\delta g^{\tau\tau}\delta^{(3)}R + \frac{1}{2}\lambda_1(r)^\mu_\nu\delta g^{\tau\tau}\delta K^\nu_\mu + \frac{1}{2}\mathcal{M}_1^2(r)(\bar{n}^\mu\partial_\mu\delta g^{\tau\tau})^2$$
$$+ \frac{1}{2}\mathcal{M}_2^2(r)\delta K(\bar{n}^\mu\partial_\mu\delta g^{\tau\tau}) + \frac{1}{2}\mathcal{M}_3^2(r)\bar{h}^{\mu\nu}\partial_\mu\delta g^{\tau\tau}\partial_\nu\delta g^{\tau\tau}\Big]$$

- Tadpole cancellation condition

$$\Lambda - c = M_\star^2(\bar{G}^\tau{}_\rho - \bar{G}^\rho{}_\rho) \ ,$$

$$\Lambda + c + \frac{2}{r^2}\sqrt{\frac{B}{A}}\left(r^2\sqrt{1-A}\,\zeta\right)' = -M_\star^2\bar{G}^\tau{}_\tau \ ,$$

$$\left[\partial_\rho\bar{K} + \frac{1-A}{r}\left(\frac{B}{A}\right)'\right]\alpha + \frac{A'B}{2A}\alpha' + \sqrt{\frac{B(1-A)}{A}}\tilde{\beta}' = -M_\star^2\bar{G}^\tau{}_\rho \ ,$$

$$\frac{1}{2r^2}\sqrt{\frac{B}{A}}\left[r^4\sqrt{\frac{B}{A}}\left(\frac{1-A}{r^2}\right)'\alpha\right]' = M_\star^2(\bar{G}^\rho{}_\rho - \bar{G}^\theta{}_\theta) \ ,$$

$$\bar{G}^\tau{}_\tau = -\frac{[r(1-B)]'}{r^2} + \frac{1-A}{r}\left(\frac{B}{A}\right)' , \quad \bar{G}^\rho{}_\rho = -\frac{[r(1-B)]'}{r^2} - \frac{1}{r}\left(\frac{B}{A}\right)' ,$$

$$\bar{G}^\tau{}_\rho = -\frac{1-A}{r}\left(\frac{B}{A}\right)' , \quad \bar{G}^\theta{}_\theta = \frac{B(r^2A')'}{2r^2A} + \frac{(r^2A)'}{4r^2}\left(\frac{B}{A}\right)' .$$

Applications to BHs with timelike scalar profile

- Background analysis for spherical BH
  [arXiv: 2204.00228 w/ V.Yingcharoenrat]
- Odd-parity perturbation around spherical BH
  → Generalized Regge-Wheeler equation
  [arXiv: 2208.02943 w/ K.Takahashi & V.Yingcharoenrat]
  [see also arXiv: 2208.02823 by Khoury, Noumi, Trodden, Wong]

Odd-parity perturbations

- General odd-parity perturbations

$$\delta g_{\tau\tau} = \delta g_{\tau\rho} = \delta g_{\rho\rho} = 0 \ ,$$

$$\delta g_{\tau a} = \sum_{\ell,m} r^2 h_{0,\ell m}(\tau,\rho) E_a{}^b \bar{\nabla}_b Y_{\ell m}(\theta,\phi) \ ,$$

$$\delta g_{\rho a} = \sum_{\ell,m} r^2 h_{1,\ell m}(\tau,\rho) E_a{}^b \bar{\nabla}_b Y_{\ell m}(\theta,\phi) \ ,$$

$$\delta g_{ab} = \sum_{\ell,m} r^2 h_{2,\ell m}(\tau,\rho) E_{(a|}{}^c \bar{\nabla}_c \bar{\nabla}_{|b)} Y_{\ell m}(\theta,\phi) \ ,$$

- Gauge fixing ($\ell \geq 2$) $\quad h_2 \to 0$
- Master variable

$$\chi = \dot{h}_1 - \partial_\rho h_0 - p_4 h_1$$

- Quadratic action $S_2 = \int d\tau d\rho \; \mathcal{L}_2$

$$\frac{(j^2-2)(2\ell+1)}{2\pi j^2}\mathcal{L}_2 = s_1\dot{\chi}^2 - s_2(\partial_\rho\chi)^2 - s_3\chi^2$$

$$s_1 = \frac{j^2-2}{2\sqrt{1-A}}\frac{(M_\star^2+M_3^2)^2 r^6}{(j^2-2)M_\star^2+(M_\star^2+M_3^2)r^2p_4^2}$$

$$s_2 = \frac{(M_\star^2+M_3^2)r^6}{2(1-A)^{3/2}} \qquad j^2 \equiv \ell(\ell+1)$$

$$s_3 = j^2\frac{(M_\star^2+M_3^2)r^4}{2\sqrt{1-A}} + \mathcal{O}(j^0)$$

$$p_4 \equiv \sqrt{\frac{B}{A(1-A)}}\left(\frac{A'}{2}+\frac{1-A}{r}\right)\frac{\alpha+M_3^2}{M_\star^2+M_3^2}$$

- Sound speeds

$$c_\rho^2 = \frac{\bar{g}_{\rho\rho}}{|\bar{g}_{\tau\tau}|}\frac{s_2}{s_1} = \frac{M_\star^2}{M_\star^2 + M_3^2} + \frac{r^2 p_4^2}{j^2 - 2}$$

$$c_\theta^2 = \lim_{\ell\to\infty} \frac{r^2}{|\bar{g}_{\tau\tau}|}\frac{s_3}{j^2 s_1} = \frac{M_\star^2}{M_\star^2 + M_3^2}$$

- For p_4=0, i.e. $\alpha + M_3^2 = 0$

$$c_\rho^2 = c_\theta^2 = \frac{M_\star^2}{M_\star^2 + M_3^2} \equiv c_T^2$$

- Stability $s_1 > 0\,, \qquad c_\rho^2 > 0\,, \qquad c_\theta^2 > 0$

$$M_\star^2 + M_3^2 > 0\,, \qquad M_\star^2 > 0$$

- Going back to Schwarzschild coordinates

$$\frac{(j^2-2)(2\ell+1)}{2\pi j^2}\mathcal{L}_2 = a_1(\partial_t\chi)^2 - a_2(\partial_r\chi)^2 + 2a_3(\partial_t\chi)(\partial_r\chi) - a_4\chi^2$$

$$a_1 = \frac{s_1-(1-A)^2 s_2}{\sqrt{A^3B(1-A)}}\,, \qquad a_2 = \sqrt{\frac{B(1-A)}{A}}(s_2-s_1)\,,$$

$$a_3 = \frac{(1-A)s_2-s_1}{A}\,, \qquad a_4 = \sqrt{\frac{A}{B(1-A)}}s_3\,.$$

- Generalized Regge-Wheeler equation

$$\frac{\partial^2\Psi}{\partial\tilde{t}^2} - c_{r_*}^2\frac{\partial^2\Psi}{\partial r_*^2} + V_{\rm eff}\Psi = 0 \qquad \Psi = \sqrt{\Gamma}\,\chi$$

$$V_{\rm eff} \equiv \frac{a_4}{\tilde{a}_1} + \frac{1}{2\sqrt{AB}\,\tilde{a}_1}\frac{d^2\Gamma}{dr_*^2} - \frac{1}{4\tilde{a}_1 a_2}\left(\frac{d\Gamma}{dr_*}\right)^2 \qquad \Gamma \equiv \frac{a_2}{\sqrt{AB}}$$

$$\tilde{a}_1 = a_1 + \frac{a_3^2}{a_2}$$

$$\tilde{t} = t + \int\frac{a_3}{a_2}dr \qquad r_* = \int\frac{1}{\sqrt{AB}}dr$$

Applications to BHs with timelike scalar profile

- Background analysis for spherical BH
  [arXiv: 2204.00228 w/ V.Yingcharoenrat]
- Odd-parity perturbation around spherical BH
  → Generalized Regge-Wheeler equation
  [arXiv: 2208.02943 w/ K.Takahashi & V.Yingcharoenrat]
  [see also arXiv: 2208.02823 by Khoury, Noumi, Trodden, Wong]
  → Quasi-normal mode
  [arXiv: 2304.14304 w/ K.Takahashi & K.Tomikawa & V.Yingcharoenrat]

QNM of stealth Schwarzschild BH

[arXiv: 2304.14304 w/ K.Takahashi & K.Tomikawa & V.Yingcharoenrat]

- Background with 2m=1
  A(r) = B(r) = 1 – 1/r
- Set $p_4 = 0$ to make c_ρ^2 finite @ r→∞
- Generalized Regge-Wheeler potential

$$V_{\rm eff}(r) = (1+\alpha_{\rm T}) f(r) \left[\frac{\ell(\ell+1)}{r^2} - \frac{3r_g}{r^3} \right] \qquad f(r) = 1 - r_g/r$$

$$\alpha_{\rm T} \equiv c_{\rm T}^2 - 1 = \alpha/(M_\star^2 - \alpha) \qquad r_g \equiv r_{\rm H}/(1+\alpha_{\rm T})$$

- QNM frequency

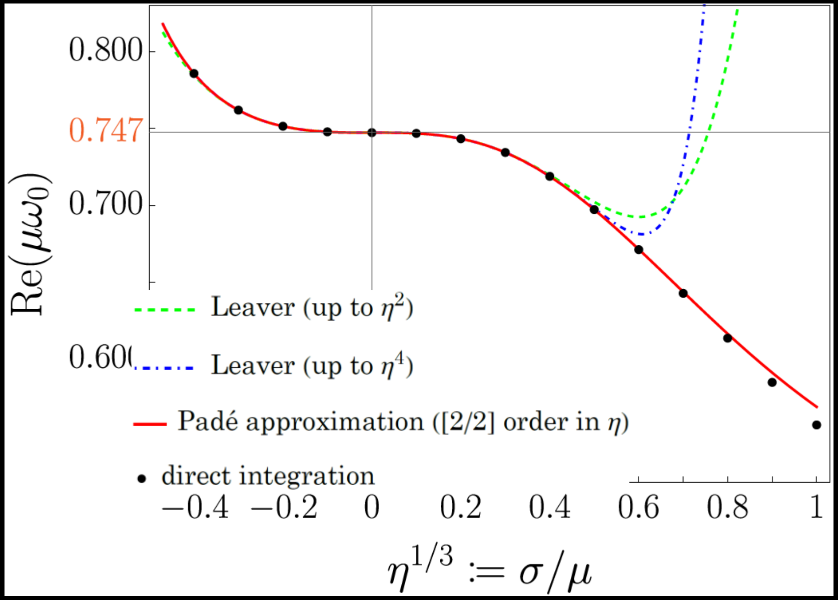
$$\omega = \omega_{\rm GR}(1+\alpha_{\rm T})^{3/2}$$

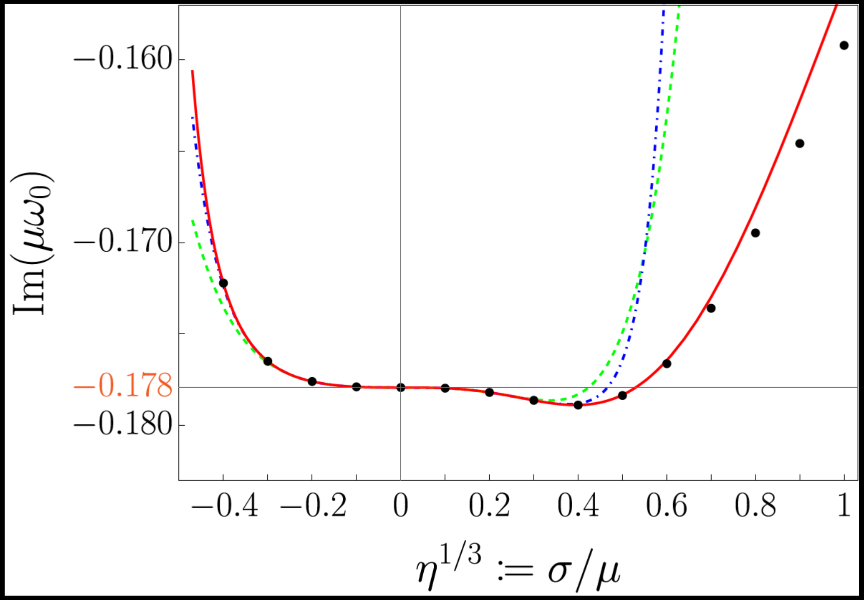
$$\to \omega_{\rm GR} \quad (c_T^2 \to 1)$$

QNM of Hayward BH

[arXiv: 2304.14304 w/ K.Takahashi & K.Tomikawa & V.Yingcharoenrat]

- Non-singular BH background $$A = B = 1 - \frac{\mu r^2}{r^3 + \sigma^3}$$
- Set $p_4 = 0$ to make c_ρ^2 finite @ $r \rightarrow \infty$
- Set $M_3^2 = 0$ to ensure $c_T^2 = 1$
- QNM frequency





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  [work in progress w/ K.Takahashi & V.Yingcharoenrat]
- Tidal Love number of spherical BH
  [work in progress w/ C.GharibAliBarura & H.Kobayashi & N.Oshita & K.Takahashi & V.Yingcharoenrat]
- Future works include Rotating BH, BH with scalar accretion [c.f. arXiv:1304.6287 by Chadburn & Gregory; arXiv:1804.03462 by Gregory, Kastor & Traschen], BH formation, etc…

EFT of scalar-tensor gravity with timelike scalar profile

SUMMARY

- Ghost condensation universally describes all scalar-tensor theories of gravity with timelike scalar profile on Minkowski background respecting time translation / reflection symmetry (up to shift / reflection of the scalar).
- Extension of ghost condensation to FLRW backgrounds results in the EFT of inflation/DE.
- These EFTs provide universal descriptions of all scalar-tensor theories of gravity with timelike scalar profile on each background, including Horndeski theory, DHOST theory, U-DHOST theory, and more.

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- If we want to learn something about the EFT of DE from BH then we need to consider BH solutions with timelike scalar profile.
- EFT of scalar-tensor gravity with timelike scalar profile on arbitrary background was developed. Consistency relations among EFT coefficients ensure the spatial diffeo invariance. Applicable to BHs with scalar field DE.

EFT of scalar-tensor gravity with timelike scalar profile

- **Time diffeo is broken by the scalar profile but spatial diffeo is preserved.**
- All terms that respect spatial diffeo must be included in the EFT action.
- Derivative & perturbative expansions
- Diffeo can be restored by introducing NG boson

EFT on Minkowski background = ghost condensation

Arkani-Hamed, Cheng, Luty and Mukohyama, JHEP 0405:074,2004

EFT on cosmological background = EFT of inflation/dark energy

Creminelli, Luty, Nicolis and Senatore 2006

Cheung, Creminelli, Fitzpatrick, Kaplan and Senatore 2007

EFT on arbitrary background = EFT of BH perturbations

Mukohyama and Yingcharoenrat, JCAP 09 (2022) 010

Taylor expansion of the general action $S = \int d^4x\sqrt{-g}\; F(R_{\mu\nu\alpha\beta}, g^{\tau\tau}, K_{\mu\nu}, \nabla_\nu, \tau)$

$$S = \int d^4x\sqrt{-g}\left[\bar{F} + \bar{F}_{g^{\tau\tau}}\delta g^{\tau\tau} + \bar{F}_K\delta K + \quad \dots\right]$$

Consistency relations ← S is invariant under spatial diffeo but the background breaks it.

$$\frac{d}{dx^i}\bar{F} = \bar{F}_{g^{\tau\tau}}\frac{\partial\bar{g}^{\tau\tau}}{\partial x^i} + \bar{F}_K\frac{\partial\bar{K}}{\partial x^i} + \dots$$

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- These EFTs provide universal descriptions of all scalar-tensor theories of gravity with timelike scalar profile on each background, including Horndeski theory, DHOST theory, U-DHOST theory, and more.
- If we want to learn something about the EFT of DE from BH then we need to consider BH solutions with timelike scalar profile.
- EFT of scalar-tensor gravity with timelike scalar profile on arbitrary background was developed. Consistency relations among EFT coefficients ensure the spatial diffeo invariance. Applicable to BHs with scalar field DE.
- Other applications? Further extensions?

Further extension of the web of EFTs

"The Effective Field Theory of Vector-Tensor Theories"
Katsuki Aoki, Mohammad Ali Gorji, Shinji Mukohyama, Kazufumi Takahashi, , JCAP 01 (2022) 01, 059 [arXiv: 2111.08119].

Residual symmetry in the unitary gauge

$$\vec{x} \to \vec{x}'(t, \vec{x})$$

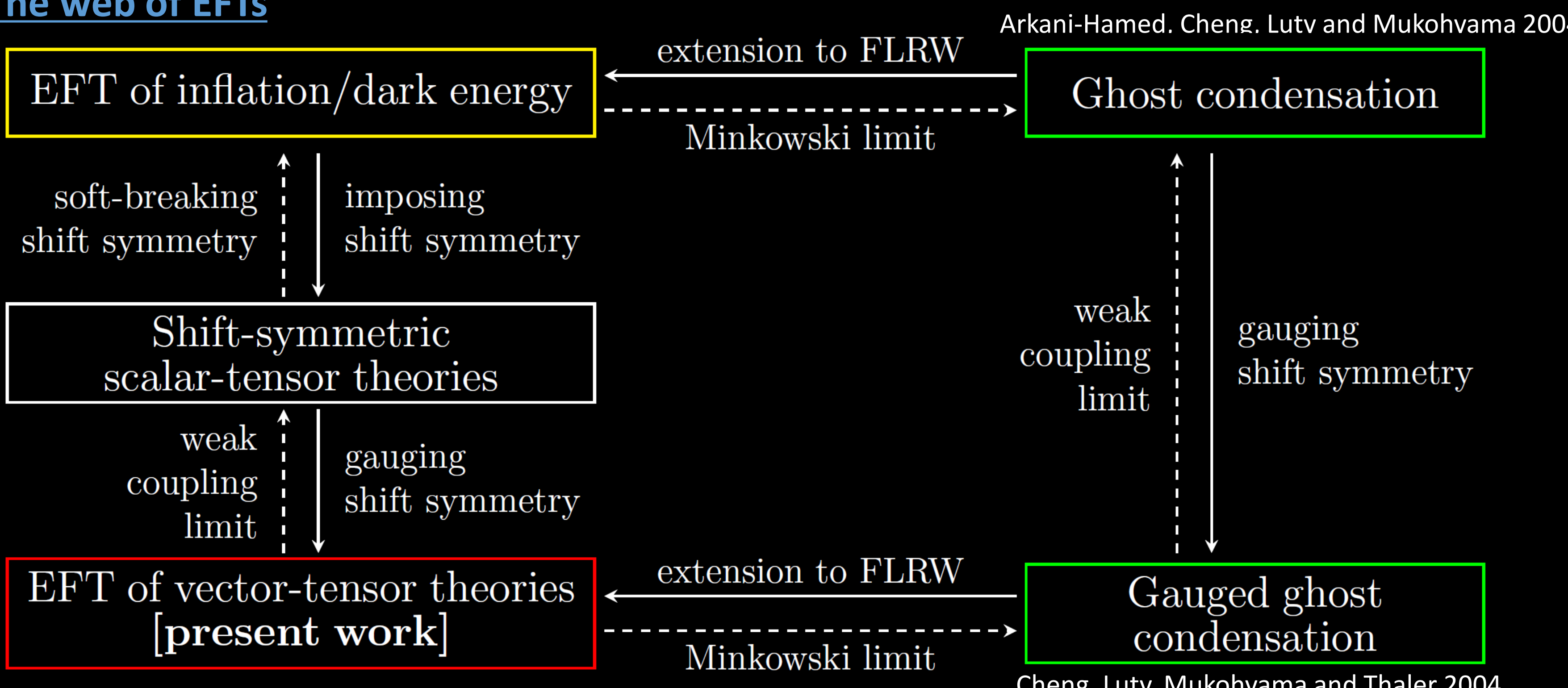
$$t \to t - g_M \chi(x)\,, \quad A_\mu \to A_\mu + \partial_\mu \chi(x)$$

leaving $\tilde{\delta}^0{}_\mu = \delta^0{}_\mu + g_M A_\mu$ invariant

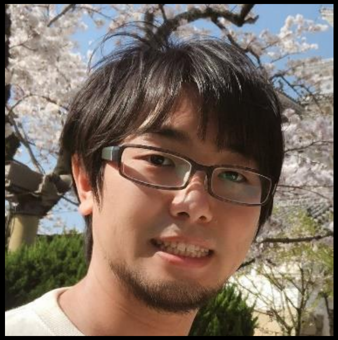
c.f. Residual symmetry in unitary gauge for scalar-tensor theories

$$\vec{x} \to \vec{x}'(t, \vec{x})$$

The web of EFTs



Thank you!



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arXiv: 2111.08119 w/ K.Aoki, M.A.Gorji & K.Takahashi

Also Arkani-Hamed, Cheng, Luty and Mukohyama 2004 (hep-th/0312099)
Mukohyama 2005 (hep-th/0502189)

EFT of ghost condensation =

EFT of scalar-tensor gravity with timelike scalar profile on Minkowski background

Arkani-Hamed, Cheng, Luty and Mukohyama 2004

Backgrounds characterized by

- ✧ $\langle \partial_\mu \phi \rangle = const \neq 0$ and timelike
- ✧ Minkowski metric

t → t + const & t → -t unbroken

up to ϕ → ϕ + const & ϕ → -ϕ

➡

$$L_{eff} = L_{EH} + M^4 \left\{ \left(h_{00} - 2\dot{\pi} \right)^2 - \frac{\alpha_1}{M^2} \left(K + \vec{\nabla}^2 \pi \right)^2 - \frac{\alpha_2}{M^2} \left(K^{ij} + \vec{\nabla}^i \vec{\nabla}^j \pi \right) \left(K_{ij} + \vec{\nabla}_i \vec{\nabla}_j \pi \right) + \cdots \right\}$$

Gauge choice: $\phi(t,\vec{x}) = t.$ $\pi \equiv \delta\phi = 0$

(Unitary gauge)

Residual symmetry: $\vec{x} \to \vec{x}'(t,\vec{x})$

⇒ Write down most general action invariant under this residual symmetry.

(⇒ Action for π: undo unitary gauge!)

Start with flat background $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

Under residual ξ^i

$$\delta h_{00} = 0, \delta h_{0i} = \partial_0 \xi_i, \delta h_{ij} = \partial_i \xi_j + \partial_j \xi_i$$

Action invariant under ξ^i

Beginning at quadratic order, since we are assuming flat space is good background.

$$\begin{cases} (h_{00})^2 & \textbf{OK} \\ \cancel{(h_{0i})^2} & \\ K^2, K^{ij}K_{ij} & \textbf{OK} \end{cases}$$

$$K_{ij} = \frac{1}{2}\left(\partial_0 h_{ij} - \partial_j h_{0i} - \partial_i h_{0j}\right)$$

$$\Rightarrow \quad L_{eff} = L_{EH} + M^4\left\{(h_{00})^2 - \frac{\alpha_1}{M^2}K^2 - \frac{\alpha_2}{M^2}K^{ij}K_{ij} + \cdots\right\}$$

Action invariant under ξ^i

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Action for π

$$\xi^0 = \pi \qquad \begin{cases} h_{00} \to h_{00} - 2\partial_0 \pi \\ K_{ij} \to K_{ij} + \partial_i \partial_j \pi \end{cases}$$

$$\Rightarrow \quad L_{eff} = L_{EH} + M^4 \left\{ (h_{00} - 2\dot{\pi})^2 - \frac{\alpha_1}{M^2}\left(K + \vec{\nabla}^2 \pi\right)^2 - \frac{\alpha_2}{M^2}\left(K^{ij} + \vec{\nabla}^i \vec{\nabla}^j \pi\right)\left(K_{ij} + \vec{\nabla}_i \vec{\nabla}_j \pi\right) + \cdots \right\}$$

$$E \to rE$$

$$dt \to r^{-1} dt$$

$$dx \to r^{-1/2} dx$$

$$\pi \to r^{1/4} \pi$$

Make invariant $\Rightarrow \int dt d^3x \left[\frac{1}{2} \dot{\pi}^2 - \frac{\alpha (\vec{\nabla}^2 \pi)^2}{M^2} + \cdots \right]$

Leading nonlinear operator in infrared $\int dt d^3x \frac{\dot{\pi}(\nabla \pi)^2}{\tilde{M}^2}$

has scaling dimension 1/4. **(Barely) irrelevant**

$\Rightarrow$ Good low-E effective theory
Robust prediction

e.g. Ghost inflation [Arkani-hamed, Creminelli, Mukohyama, Zaldarriaga 2004]

Extension to FLRW background = EFT of inflation/dark energy

Creminelli, Luty, Nicolis, Senatore 2006
Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore 2007

- Action invariant under $x^i \rightarrow x^i(t,x)$
- Ingredients

  $g_{\mu\nu}, g^{\mu\nu}, R_{\mu\nu\rho\sigma}, \nabla_\mu,$ t & its derivatives

- 1^{st} derivative of t

$$\partial_\mu t = \delta^0_\mu \qquad n_\mu = \frac{\partial_\mu t}{\sqrt{-g^{\mu\nu}\partial_\mu t \partial_\nu t}} = \frac{\delta^0_\mu}{\sqrt{-g^{00}}}$$

$$g^{00} \qquad h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$$

- 2^{nd} derivative of t

$$K_{\mu\nu} \equiv h^\rho_\mu \nabla_\rho n_\nu$$

Unitary gauge action

$$I = \int d^4x\sqrt{-g}\,L(t, \delta^0_\mu, K_{\mu\nu}, g_{\mu\nu}, g^{\mu\nu}, \nabla_\mu, R_{\mu\nu\rho\sigma})$$

derivative & perturbative expansions

$$\begin{aligned} I &= M_{Pl}^2 \int dx^4\sqrt{-g}\left[\frac{1}{2}R + c_1(t) + c_2(t)g^{00}\right. \\ &\left. \quad + L^{(2)}(\tilde{\delta}g^{00}, \tilde{\delta}K_{\mu\nu}, \tilde{\delta}R_{\mu\nu\rho\sigma}; t, g_{\mu\nu}, g^{\mu\nu}, \nabla_\mu)\right] \\ L^{(2)} &= \lambda_1(t)(\tilde{\delta}g^{00})^2 + \lambda_2(t)(\tilde{\delta}g^{00})^3 + \lambda_3(t)\tilde{\delta}g^{00}\tilde{\delta}K^\mu_\mu \\ &\quad + \lambda_4(t)(\tilde{\delta}K^\mu_\mu)^2 + \lambda_5(t)\tilde{\delta}K^\mu_\nu\tilde{\delta}K^\nu_\mu + \cdots \end{aligned}$$

$$\tilde{\delta}g^{00} \equiv g^{00} + 1 \qquad \tilde{\delta}K_{\mu\nu} \equiv K_{\mu\nu} - H\gamma_{\mu\nu}$$

$$\tilde{\delta}\mathrm{R}_{\mu\nu\rho\sigma} \equiv \mathrm{R}_{\mu\nu\rho\sigma} - 2(H^2 + \mathfrak{K}/a^2)\gamma_{\mu[\rho}\gamma_{\sigma]\nu} + (\dot{H} + H^2)(\gamma_{\mu\rho}\delta^0_\nu\delta^0_\sigma + (3\mathrm{perm.}))$$

NG boson

- Undo unitary gauge $t \to \tilde{t} = t - \pi(\tilde{t}, \vec{x})$

$$
\begin{aligned}
H(t) &\to H(t+\pi), \quad \dot{H}(t) \to \dot{H}(t+\pi), \\
\lambda_i(t) &\to \lambda_i(t+\pi), \quad a(t) \to a(t+\pi), \\
\delta^0_\mu &\to (1+\dot{\pi})\delta^0_\mu + \delta^i_\mu \partial_i \pi,
\end{aligned}
$$

- NG boson in decoupling (subhorizon) limit

$$
\begin{aligned}
I_\pi = M_{Pl}^2 \int dt d^3\vec{x}\, a^3 \Bigg\{ &-\frac{\dot{H}}{c_s^2}\left(\dot{\pi}^2 - c_s^2 \frac{(\partial_i \pi)^2}{a^2}\right) \\
&-\dot{H}\left(\frac{1}{c_s^2} - 1\right)\left(\frac{c_3}{c_s^2}\dot{\pi}^3 - \dot{\pi}\frac{(\partial_i \pi)^2}{a^2}\right) + O(\pi^4, \tilde{\epsilon}^2) + L^{(2)}_{\tilde{\delta}K, \tilde{\delta}R} \Bigg\}
\end{aligned}
$$

$$
\frac{1}{c_s^2} = 1 - \frac{4\lambda_1}{\dot{H}}, \quad c_3 = c_s^2 - \frac{8c_s^2\lambda_2}{-\dot{H}}\left(\frac{1}{c_s^2} - 1\right)^{-1}
$$

- Sound speed

c_s : speed of propagation for modes with $\omega \gg H$

$\omega^2 \simeq c_s^2 \frac{k^2}{a^2}$ for $\pi \sim A(t) \exp\left(-i\int \omega dt + i\vec{k}\cdot\vec{x}\right)$

It is not straightforward…

- General action in the unitary gauge ($\phi = \tau$)

$$S = \int d^4x \sqrt{-g}\, F(R_{\mu\nu\alpha\beta}, g^{\tau\tau}, K_{\mu\nu}, \nabla_\nu, \tau)$$

- Taylor expansion around the background

$$S = \int d^4x \sqrt{-g} \left[\bar{F} + \bar{F}_{g^{\tau\tau}} \delta g^{\tau\tau} + \bar{F}_K \delta K + \cdots \right]$$

- The whole action is invariant under 3d diffeo but **each term is not…**
- Each coefficient is a function of (τ, x^i) but cannot be promoted to an arbitrary function.

Solution: consistency relations

- The chain rule

$$\left\{ \begin{array}{l} \dfrac{d}{dx^i}\bar{F} = \bar{F}_{g^{\tau\tau}}\dfrac{\partial \bar{g}^{\tau\tau}}{\partial x^i} + \bar{F}_K \dfrac{\partial \bar{K}}{\partial x^i} + \dots \\ \dfrac{d}{dx^i}\bar{F}_{g^{\tau\tau}} = \bar{F}_{g^{\tau\tau}g^{\tau\tau}}\dfrac{\partial \bar{g}^{\tau\tau}}{\partial x^i} + \bar{F}_{g^{\tau\tau}K}\dfrac{\partial \bar{K}}{\partial x^i} + \dots \\ \dfrac{d}{dx^i}\bar{F}_K = \bar{F}_{g^{\tau\tau}K}\dfrac{\partial \bar{g}^{\tau\tau}}{\partial x^i} + \bar{F}_{KK}\dfrac{\partial \bar{K}}{\partial x^i} + \dots \end{array} \right.$$

  relates x^i-derivatives of an EFT coefficient to other EFT coefficients, and leads to consistency relations.
- **The consistency relations ensure the spatial diffeo invariance.**
- Taylor coefficients should satisfy the consistency relations but are otherwise arbitrary.
- (No consistency relation for τ-derivatives.)

EFT action

$$S = \int d^4x\sqrt{-g}\Big[\frac{M_\star^2}{2}f(y)R - \Lambda(y) - c(y)g^{\tau\tau} - \beta(y)K - \alpha^\mu_\nu(y)\sigma^\nu_\mu - \gamma^\mu_\nu(y)r^\nu_\mu + \frac{1}{2}m_2^4(y)(\delta g^{\tau\tau})^2$$
$$+ \frac{1}{2}M_1^3(y)\delta g^{\tau\tau}\delta K + \frac{1}{2}M_2^2(y)\delta K^2 + \frac{1}{2}M_3^2(y)\delta K^\mu_\nu \delta K^\nu_\mu + \frac{1}{2}M_4(y)\delta K\delta^{(3)}R$$
$$+ \frac{1}{2}M_5(y)\delta K^\mu_\nu\delta^{(3)}R^\nu_\mu + \frac{1}{2}\mu_1^2(y)\delta g^{\tau\tau}\delta^{(3)}R + \frac{1}{2}\mu_2(y)\delta^{(3)}R^2 + \frac{1}{2}\mu_3(y)\delta^{(3)}R^\mu_\nu\delta^{(3)}R^\nu_\mu$$
$$+ \frac{1}{2}\lambda_1(y)^\nu_\mu\delta g^{\tau\tau}\delta K^\mu_\nu + \frac{1}{2}\lambda_2(y)^\nu_\mu\delta g^{\tau\tau}\delta^{(3)}R^\mu_\nu + \frac{1}{2}\lambda_3(y)^\nu_\mu\delta K\delta K^\mu_\nu + \frac{1}{2}\lambda_4(y)^\nu_\mu\delta K\delta^{(3)}R^\mu_\nu$$
$$+ \frac{1}{2}\lambda_5(y)^\nu_\mu\delta^{(3)}R\delta K^\mu_\nu + \frac{1}{2}\lambda_6(y)^\nu_\mu\delta^{(3)}R\delta^{(3)}R^\mu_\nu + \dots\Big],$$

- EFT coefficients should satisfy the consistency relations but are otherwise arbitrary
- One can restore 4d diffeo by Stueckelberg trick
- Easy to find dictionary between EFT coefficients and theory parameters
- Can be applied to BH with timelike scalar profile
- Bridge between theories and observations

Stealth solutions in k-essence

Mukohyama 2005

- Action in Einstein frame

$$I = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R + P(X)\right] \qquad X = -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

- EOMs

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}P'(X)g^{\mu\nu}\partial_\nu\phi\right) = 0$$

$$M_{\rm Pl}^2 G_{\mu\nu} = 2P'(X)\partial_\mu\phi\partial_\nu\phi + P(X)g_{\mu\nu}$$

- Stealth sol with $X = X_0$, where $P'(X_0)=0$

$$G_{\mu\nu} = \Lambda_{\rm eff}\, g_{\mu\nu} \qquad \Lambda_{\rm eff} = P(X_0)/M_{\rm Pl}^2$$

- $X = X_0$ ($\neq 0$)

⟺ $u^\mu = g^{\mu\nu}\partial_\nu\phi$ defines geodesic congruence

$$(u^\nu\nabla_\nu u^\mu = -\nabla^\mu X/2 = 0)$$

⟺ $\phi/\sqrt{|X_0|}$ **defines Gaussian normal coord.**

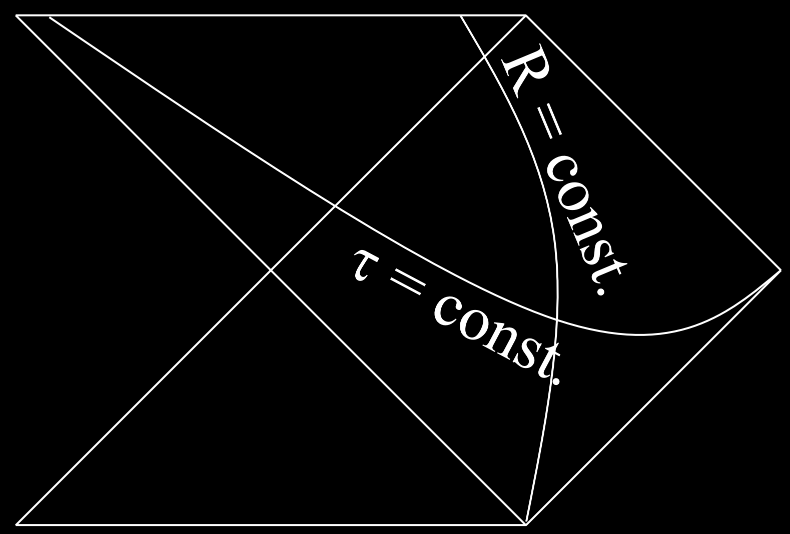
Stealth solutions in k-essence

Mukohyama 2005

- Any metric locally admits Gaussian normal coord.
- If P'(X) has a real root X_0 then any vacuum GR sol with $\Lambda_{\rm eff} = P(X_0)/M_{\rm Pl}^2$ locally leads to a stealth sol.
- Schwarzshild metric admits a "globally" well-behaved Gaussian normal coord. (Lemeitre reference frame)

$$g_{\mu\nu}dx^\mu dx^\nu = -d\tau^2 + \frac{r_g dR^2}{r(\tau,R)} + r^2(\tau,R)d\Omega^2$$

$$r(\tau,R) = \left[\frac{3}{2}\sqrt{r_g}(R-\tau)\right]^{2/3}$$



- Stealth Schwarzschild solution with $\phi = \sqrt{X_0}\tau$, if P'(X) has a positive root X_0 and if $\Lambda_{\rm eff}$ is canceled by $\Lambda_{\rm bare}$

Stealth solutions with $\phi = qt+\psi(r)$

- Schwarzschild in k-essence (Mukohyama 2005)
- Schwarzschild-dS in Horndeski theory (Babichev & Charmousis 2013, Kobayashi & Tanahashi 2014) Schwarzshild-dS in DHOST (Ben Achour & Liu 2019, Motohashi & Minamitsuji 2019)
- Kerr-dS in DHOST (Charmousis & Crisotomi & Gregory & Stergioulas 2019)
- However, perturbations around most of those stealth solutions are infinitely strongly coupled (de Rham & Zhang 2019) . This means the solutions cannot be trusted.
- Approximately stealth solution in ghost condensate does not suffer from strong coupling (Mukohyama 2005). Why?

Origin of strong coupling

- EFT around stealth Minkowski sol. (= ghost condensate) → universal dispersion relation without the usual k^2 term

$$\omega^2 = \alpha k^4 / M^2$$

- For α = O(1) (>0), EFT is weakly coupled all the way up to ~M. [$E_{\rm cubic} \simeq |\alpha|^{7/2} M$]
- If eom's for perturbations are strictly 2nd order (as in DHOST) then α = 0 and the dispersion relation loses dependence on k → strong coupling
- [For $\omega^2 = c_s^2 k^2$, strong coupling @ E~ $c_s^{7/4} M$]

A solution: scordatura

Motohashi & Mukohyama 2019

- Detuning of degeneracy condition recovers $\omega^2 = \alpha k^4/M^2$ and uplifts the strong coupling scale to ~$|\alpha|^{7/2}M$. If the amount of detuning is small enough then an apparent ghost is heavy enough to be integrated out.
- Scordatura = weak and controlled detuning of degeneracy condition
- Scordatura DHOST realizes ghost condensation near stealth solutions while it behaves as DHOST away from them.

Strong coupling scales

- EFT of inflation/DE in decoupling limit

$$S_\pi = M_{\rm Pl}^2 \int dt d^3\vec{x}\, a^3 \left[-\frac{\dot{H}}{c_{\rm s}^2}\left(\dot{\pi}^2 - c_{\rm s}^2 \frac{(\partial_i \pi)^2}{a^2}\right) - \dot{H}\left(\frac{1}{c_{\rm s}^2} - 1\right)\left(\frac{c_3}{c_{\rm s}^2}\dot{\pi}^3 - \dot{\pi}\frac{(\partial_i\pi)^2}{a^2}\right) + \mathcal{O}(\pi^4, \tilde{\epsilon}^2) + \mathcal{L}^{(2)}_{\tilde{\delta K}, \tilde{\delta R}} \right]$$

$$\frac{1}{c_{\rm s}^2} = 1 + \frac{4\lambda_1}{-\dot{H}}, \quad c_3 = c_{\rm s}^2 - \frac{8c_{\rm s}^2\lambda_2}{-\dot{H}}\left(\frac{1}{c_{\rm s}^2} - 1\right)^{-1}$$

- If $c_s^2 \simeq$ const is not too small, $\mathcal{L}^{(2)}_{\tilde{\delta K}, \tilde{\delta R}}$ can be ignored. We further assume $0 < c_s < 1$.

$$S_\pi = \int dt d^3\vec{\tilde{x}}\, a^3 (c_{\rm s}\epsilon M_{\rm Pl}^2 H^2)\left[\dot{\pi}^2 - \frac{(\tilde{\partial}_i\pi)^2}{a^2} + \left(\frac{1}{c_{\rm s}^2} - 1\right)\dot{\pi}\left(c_3\dot{\pi}^2 - \frac{(\tilde{\partial}_i\pi)^2}{a^2}\right) + \cdots\right]$$

$$\vec{x} = c_{\rm s}\vec{\tilde{x}}$$

$$\dot{\pi}^2 \sim \frac{(\tilde{\partial}_i\pi)^2}{a^2} \sim \frac{E^4}{c_{\rm s}\epsilon M_{\rm Pl}^2 H^2} \qquad \left(\frac{1}{c_{\rm s}^2} - 1\right)|\dot{\pi}|\Big|_{E=E_{\rm cubic}} \sim \frac{1}{\max[|c_3|, 1]}$$

➡ $$E_{\rm cubic} \lesssim \frac{(c_{\rm s}^5 \epsilon M_{\rm Pl}^2 H^2)^{1/4}}{\sqrt{1 - c_{\rm s}^2}} \to 0 \quad (c_{\rm s}^5\epsilon/(1 - c_{\rm s}^2)^2 \to 0)$$

Strong coupling scales

- De Sitter limit = small c_s^2 limit

$$S_\pi = M_{\rm Pl}^2 \int dt d^3\vec{x}\, a^3 \left[4\lambda_1 \left(\dot\pi^2 - c_{\rm s}^2 \frac{(\partial_i \pi)^2}{a^2} - \dot\pi \frac{(\partial_i\pi)^2}{a^2} \right) + 4(\lambda_1 - 2\lambda_2)\dot\pi^3 + \lambda_3 \left(H - \frac{\partial_j^2 \pi}{a^2} \right) \frac{(\partial_i\pi)^2}{a^2} + (\lambda_4 + \lambda_5) \frac{(\partial_i^2\pi)^2}{a^4} + \cdots \right]$$

$$\lambda_1 = \frac{M^4}{8M_{\rm Pl}^2}, \quad \lambda_3 = \frac{M^3\beta}{2M_{\rm Pl}^2}, \quad \lambda_4 = -\frac{M^2(\alpha+\gamma)}{2M_{\rm Pl}^2}, \quad \lambda_5 = \frac{M^2\gamma}{2M_{\rm Pl}^2}$$

$$S_\pi = \frac{M^4}{2} \int dt d^3\vec{x}\, a^3 \left[\dot\pi^2 - c_{\rm s}^2 \frac{(\partial_i\pi)^2}{a^2} - \dot\pi \frac{(\partial_i\pi)^2}{a^2} - \frac{\alpha}{M^2} \frac{(\partial_i^2\pi)^2}{a^4} + \frac{\beta}{M} \left(H - \frac{\partial_j^2\pi}{a^2} \right) \frac{(\partial_i\pi)^2}{a^2} + \cdots \right]$$

$$E^{-1} p^{-3} M^4 (E\pi)^2 \sim 1 \quad \Rightarrow \quad \pi \sim \frac{E^{3/2}}{p^{1/2} M^2}$$

$$\left. \frac{E\pi p^2}{E^2} \right|_{E=E_{\rm cubic}} \sim 1 \quad \Rightarrow \quad \left(\frac{p}{E}\right)^{7/4} \left. \frac{E}{M} \right|_{E=E_{\rm cubic}} \sim 1$$

$$\frac{\omega^2}{M^2} = \alpha \frac{k^4}{M^4 a^4} \quad \text{for} \quad \max\left[c_{\rm s}^2, |\beta| \frac{H}{M} \right] \ll |\alpha| \frac{k^2}{M^2 a^2} \ll 1$$

$$\Rightarrow \quad E_{\rm cubic} \simeq |\alpha|^{7/2} M$$

Approximately stealth BH in ghost condensate

Mukohyama 2005

- Two time scales: $t_{BH} << t_{GC} \sim M_{Pl}^2/M^3$
- For $t_{BH} << t << t_{GC}$, a usual BH sol is a good approximation → approximately stealth

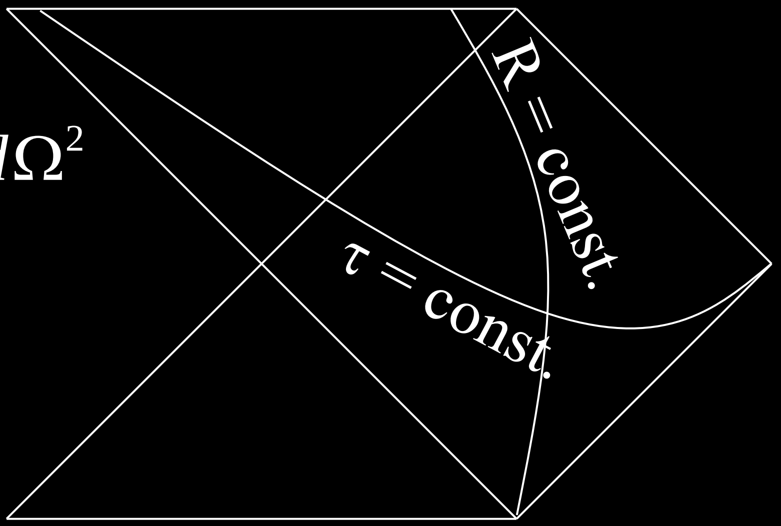
Schwarzschild metric:

$$g_{\mu\nu}dx^\mu dx^\nu = -d\tau^2 + \frac{r_g dR^2}{r(\tau,R)} + r^2(\tau,R)d\Omega^2$$

$$r(\tau,R) = \left[\frac{3}{2}\sqrt{r_g}(R-\tau)\right]^{2/3}$$

$$E = -\xi^\mu p_\mu \qquad \xi^\mu = \partial_\tau + \partial_R$$

$\phi = M^2\tau$ ➡ Exact sol in the absence of higher derivative terms



Approximately stealth BH in ghost condensate

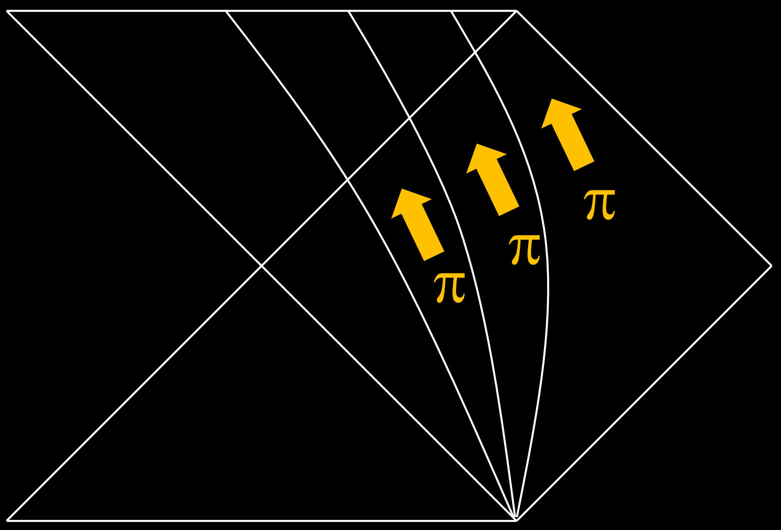
Mukohyama 2005; Cheng, Luty, Mukohyama and Thaler 2006

- A tiny tadpole due to higher derivative terms is canceled by extremely slow time-dependence.
- As a result, $\pi = \delta\phi$ starts accreting gradually.
- XTE J1118+480 (M_{bh}~$7M_{sun}$,r~$3R_{sun}$,t~240Myr or 7 Gyr) ➡ $M<10^{12}$GeV much weaker than M<100GeV

$$M_{bh} = M_{bh0} \times \left[1 + \frac{9\alpha M^2}{4M_{Pl}^2} \left(\frac{3M_{Pl}^2 v}{4M_{bh0}} \right) \right]^{2/3}$$

v : advanced null coordinate

α : coefficient of h.d. term



See DeFelice, Mukohyama, Takahashi, arXiv: 2212.13031 for a similar formula in more general HOST.

Summary of stealth BH with timelike scalar profile

- Stealth solutions = backgrounds with GR metric and non-trivial scalar profile → examples of BH solutions with timelike scalar profile
- They suffer from strong coupling problem, which is solved by scordatura (= controlled detuning of degeneracy condition)
- DHOST/Horndeski do not include scordatura but U-DHOST does (DeFelice, Mukohyama, Takahashi 2022) .
- EFT of ghost condensation already included scordatura.

- Approximately stealth solutions in ghost condensation (Mukohyama 2005) and in more general HOST with scordatura (DeFelice & Mukohyama & Takahashi, arXiv: 2212.13031) are stealth at astrophysical scales (no need for screening?, c.f. arXiv:1402.4737 by Davis, Gregory, Jha & Muir) and are free from the strong coupling problem.