

Domain walls, nHz gravitational waves, and a bit of dark matter

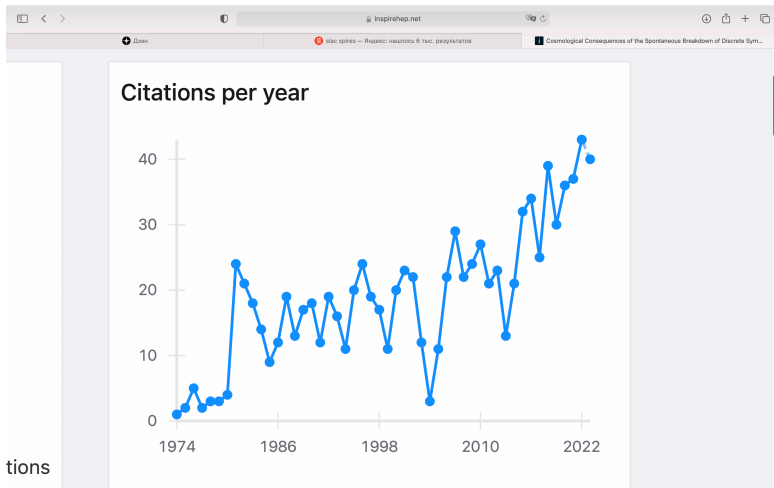
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CAS-JSPS-IBS CTPU-CGA workshop
Prague

In collaboration with
E. Babichev, D. Gorbunov, R. Samanta, A. Vikman

12 October 2023

Cosmic domain walls Zeldovich, Kobzarev, Okun'74



Chapter 1.

(Ab)normal domain walls,
or constant tension domain walls

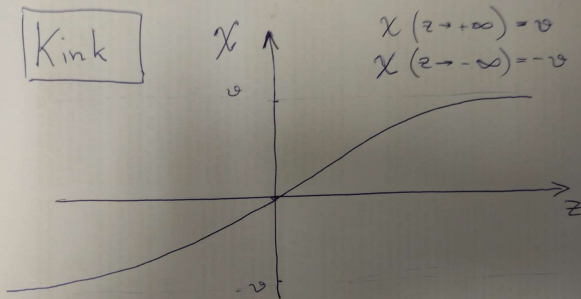
Domain walls arise in models with spontaneous breaking of discrete symmetries, e.g., Z_2

$$\mathcal{L} = \frac{(\partial_\mu \chi)^2}{2} - \frac{\lambda \cdot (\chi^2 - v^2)^2}{4}$$

Static localized solution in 1 + 1D

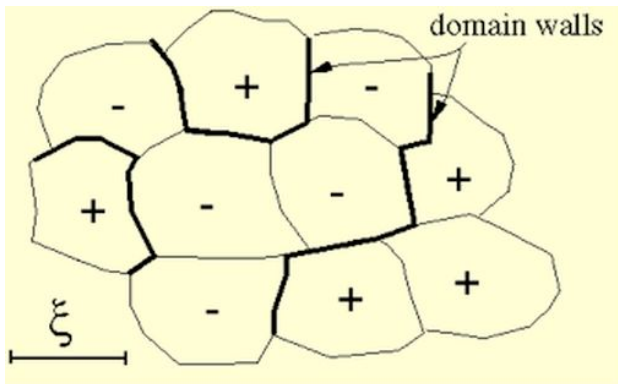
Kink $\chi(z) = v \cdot \tanh \left(\sqrt{\frac{\lambda}{2}} \cdot v \cdot z \right)$

Kink



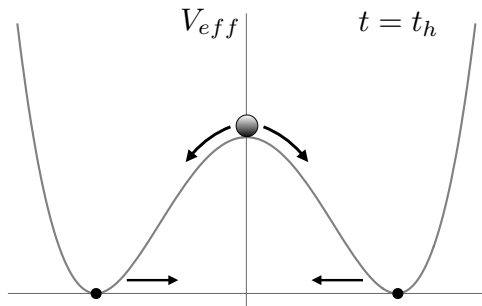
Domain walls are embeddings of kinks into 4D

Domain walls separate regions, where $\chi = \pm v$



The picture is taken from <http://www.ctc.cam.ac.uk/>

Domain walls are formed through Kibble-Zurek mechanism.



Domain wall problem

In the scaling regime: one or a few domain walls
in the horizon volume $\sim H^{-3}$.

Ryden, Press, Spergel'89

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in the horizon volume $\sim H^{-3}$.

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$$\rho_{\text{wall}} \sim M_{\text{wall}} H^3 \sim \sigma_{\text{wall}} H$$

Domain wall tension:
$$\sigma_{\text{wall}} = \frac{M_{\text{wall}}}{S} = \frac{2\sqrt{2\lambda}v^3}{3}$$

Constant tension domain walls: $\rho_{\text{wall}} \sim \sigma_{\text{wall}} H \propto T^2$

$$\frac{\rho_{\text{wall}}}{\rho_{\text{rad}}} \propto \frac{1}{T^2(t)} \propto a^2(t) \implies \text{domain walls overclose the Universe!}$$

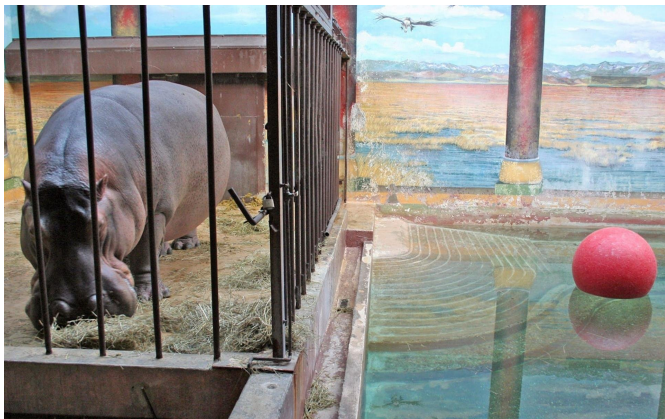
Domain walls are very energetic
and threat standard cosmological evolution.



Possible solution: explicitly break Z_2 -symmetry

$$V_{bias}(\chi) = \epsilon v \chi (\chi^2 - v^2)$$

This solution of domain wall problem looks inhuman!



Domain walls emit gravitational waves

Einstein quadrupole formula+dimensional considerations

$$\text{Power of gravitational radiation: } P \sim \frac{\ddot{Q}_{ij}\ddot{Q}_{ij}}{40\pi M_{Pl}^2}$$

$$\text{Quadrupole moment: } Q_{ij} \sim \frac{M_{wall}}{H^2} \sim \frac{\sigma_{wall}}{H^4}$$

$$\rho_{gw} \sim (P \cdot t) \cdot H^3 \sim \frac{\sigma_{wall}^2}{M_{Pl}^2} \implies \frac{\rho_{gw}}{\rho_{rad}} \propto a^4$$

Most energetic gravitational waves are emitted, when the domain wall network is being destroyed.

We are moving to Japan!

Numerical simulations: Hiramatsu, Kawasaki, Saikawa'13

$$f_{peak} \simeq H(t_{dec}) \cdot \frac{a(t_{dec})}{a_0}$$

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$$\Omega_{gw,peak} h_0^2 = \frac{\epsilon_{gw} \mathcal{A}^2}{\rho_{tot,0}} \cdot \frac{\sigma_{wall}^2}{M_{Pl}^2} \cdot \left(\frac{a(t_{dec})}{a_0} \right)^4 \quad \Omega_{gw} = \frac{d\rho_{gw}}{\rho_{tot} d \ln f} \quad \epsilon_{gw} \mathcal{A}^2 \approx 0.5$$

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$$\Omega_{gw} = \frac{d\rho_{gw}}{\rho_{tot} d \ln f} \quad \epsilon_{gw} \mathcal{A}^2 \approx 0.5$$

$$\Omega_{gw}(f) \simeq \Omega_{gw,peak} \begin{cases} \left(\frac{f}{f_{peak}} \right)^3 & f \lesssim f_{peak} \\ \frac{f_{peak}}{f} & f \gtrsim f_{peak} \end{cases}$$

Caprini et al'09 Cai, Pi, Sasaki'19

God loves '3'.



arXiv eprints submitted since Oct 5 are missing due to a technical problem. We are working with arXiv to resolve it.

The NANOGrav 15 yr Data Set: Search for Signals from New Physics

NANOGrav Collaboration · Adeela Afzal (Munster U. and Quid-i-Azam U.) [Show All\(123\)](#)

Jun 28, 2023

56 pages

Published in: *Astrophys.J.Lett.* 951 (2023) 1, L11

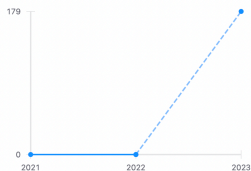
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e-Print: [2306.16219](#) [astro-ph.HE]DOI: [10.3847/2041-8213/acdc91](#)Experiments: [NANOGrav](#)View in: [ADS Abstract Service](#)

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Abstract: (IOP)

The 15 yr pulsar timing data set collected by the North American Nanohertz Observatory for Gravitational Waves (NANOGrav) shows positive evidence for the presence of a low-frequency gravitational-wave (GW) background. In this paper, we investigate potential cosmological interpretations of this signal, specifically cosmic inflation, scalar-induced GWs, first-order phase transitions, cosmic strings, and domain walls. We find that, with the exception of stable cosmic strings of field theory origin, all these models can reproduce the observed signal. When compared to the standard interpretation in terms of inspiraling supermassive black hole binaries (SMBHBs), many cosmological models seem to provide a better fit resulting in Bayes factors in the range from 10 to 100. However, these results strongly depend on modeling assumptions about the cosmic SMBHB population and, at this stage, should not be regarded as evidence for new physics. Furthermore, we identify excluded parameter regions where the predicted GW signal from cosmological sources significantly exceeds the NANOGrav signal. These parameter constraints are independent of the origin of the NANOGrav signal and illustrate how pulsar timing data provide a new way to constrain the parameter space of these models. Finally, we search for deterministic signals produced by models of ultralight dark matter (ULDM) and dark matter substructures in the Milky Way. We find no evidence for either of these signals and thus report updated constraints on these models. In the case of ULDM, these constraints outperform torsion balance and atomic clock constraints for ULDM coupled to electrons, muons, or gluons.

Note: 74 pages, 31 figures, 4 tables; published in *Astrophysical Journal Letters* as part of Focus on NANOGrav's 15-year Data Set and the Gravitational Wave Background. For questions or comments, please email comments@nanograv.org

[black hole: binary](#) [cosmic string: stability](#) [NANOGrav](#) [new physics](#) [dark matter: mass](#) [gravitational radiation](#) [pulsar](#) [galaxy](#) [observatory](#) [critical phenomena](#) [Show all \(19\)](#)[References \(322\)](#)[Figures \(44\)](#)

Devil's trap: $3 = 2$

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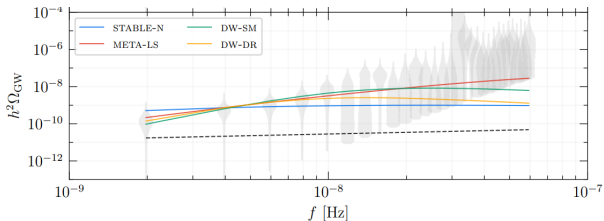
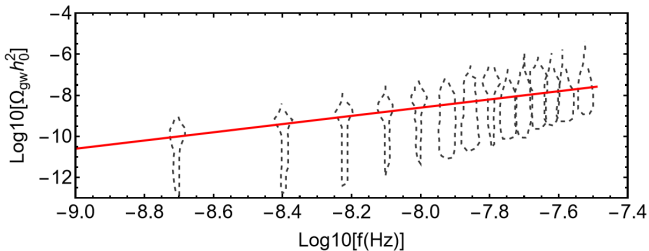
$$\Omega_{gw}(f) = \Omega_{yr} \cdot \left(\frac{f}{f_{yr}} \right)^{5-\gamma}$$

NANOGrav definition of the spectral index is different from human one!

$$\Omega_{gw}(f) \propto f^3 \implies \gamma = 2$$

$$\gamma = 3 \implies \Omega_{gw}(f) \propto f^2$$

$\gamma = 3.2 \pm 0.6$ 68% CL NANOGrav 15 yr



Still one needs to release the hyppo!



Chapter 2.

Melting domain walls.

$$\mathcal{L} = \frac{(\partial_\mu \chi)^2}{2} - \frac{\lambda(\chi^2 - v^2(T))^2}{4}$$

$$v(T) \propto T \propto \frac{1}{a(t)}$$

Something, what one could expect from **scale-invariant** physics.

No domain wall problem

$$v \propto T \implies \sigma_{\text{wall}} \sim \sqrt{\lambda} v^3 \propto T^3$$

$$\rho_{\text{wall}} \simeq \sigma_{\text{wall}} H \propto T^5 \quad \frac{\rho_{\text{wall}}}{\rho_{\text{rad}}} \propto T(t) \propto \frac{1}{a(t)}$$

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Energy density of domain walls redshifts faster than radiation

Domain walls completely vanish at inverse phase transition

Vilenkin'81

Do melting domain walls leave any trace?

Domain walls emit gravitational waves

Einstein quadrupole formula+dimensional considerations

$$\text{Power of gravitational radiation: } P \sim \frac{\ddot{Q}_{ij}\ddot{Q}_{ij}}{40\pi M_{Pl}^2}$$

$$\text{Quadrupole moment: } Q_{ij} \sim \frac{M_{wall}}{H^2} \sim \frac{\sigma_{wall}}{H^4}$$

$$\rho_{gw} \sim (P \cdot t) \cdot H^3 \sim \frac{\sigma_{wall}^2}{M_{Pl}^2} \implies \rho_{gw}(t) \propto T^6(t) \propto \frac{1}{a^6(t)}$$

Most energetic gravitational waves are emitted right after domain wall formation

$\gamma = 3$ from melting domain walls

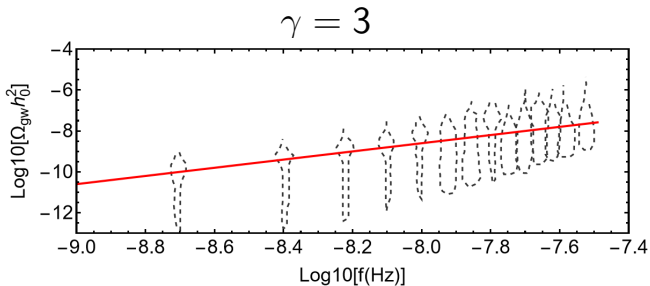
Gravitational waves produced around the time t :

$$\rho_{gw,0} = \rho_{gw}(t) \cdot \left(\frac{a(t)}{a_0} \right)^4 \propto T^2(t)$$

Characteristic present-day frequency:

$$f \simeq H(t) \cdot \frac{a(t)}{a_0} \propto T(t)$$

$$\frac{d\rho_{gw,0}}{d \ln f} \propto f^2 \implies \gamma = 3$$



- Where does $v(T) \propto T$ come from?
- What is the amplitude of GWs?

Chapter 3.

Energetic domain walls
from large N -limit of conformal field theories.

$$\mathcal{L} = \frac{(\partial_\mu \chi)^2}{2} - \frac{\lambda \cdot \chi^4}{4} + \frac{g^2 \chi^2 \phi^\dagger \phi}{2}.$$

χ is cold

ϕ is in thermal equilibrium with plasma

$$0 < g^2 \ll 1$$

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$$\langle \phi^\dagger \phi \rangle_T = \frac{NT^2}{12} \implies V_{\text{eff}} = \frac{\lambda \cdot \chi^4}{4} - \frac{Ng^2 T^2 \chi^2}{24}$$

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$T \propto \frac{1}{a(t)} \implies Z_2$ -symmetry breaking at early times

$$v^2 = \frac{Ng^2 T^2}{12\lambda}$$

Numerical simulations: Hiramatsu, Kawasaki, Saikawa'13

$$f_{peak} \simeq H(t_i) \cdot \frac{a(t_i)}{a_0}$$

$$\Omega_{gw,peak} h_0^2 = \frac{\epsilon_{gw} \mathcal{A}^2}{\rho_{tot,0}} \cdot \frac{\sigma_{wall}^2(t_i)}{M_{Pl}^2} \cdot \left(\frac{a(t_i)}{a_0} \right)^4$$

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$$f_{peak} \simeq 6 \text{ nHz} \cdot \sqrt{\frac{N}{B}} \cdot \left(\frac{g}{10^{-18}} \right)$$

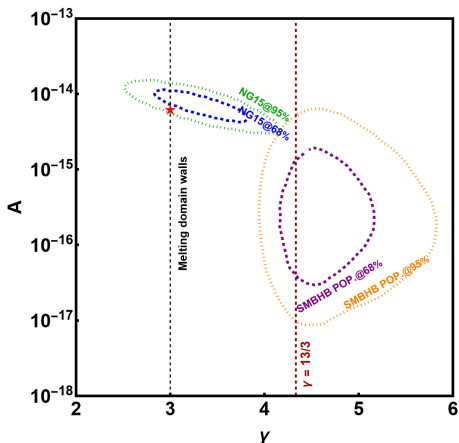
$$\Omega_{gw,peak} \cdot h_0^2 \approx \frac{4 \cdot 10^{-14} \cdot N^4}{B \cdot \beta^2}$$

$B = \ln^2 \frac{2 \langle \chi \rangle}{\delta \chi} \simeq 1 - 100$ contains info about domain wall formation

Vanilla region: $\beta \equiv \frac{\lambda}{g^4} \simeq 1$

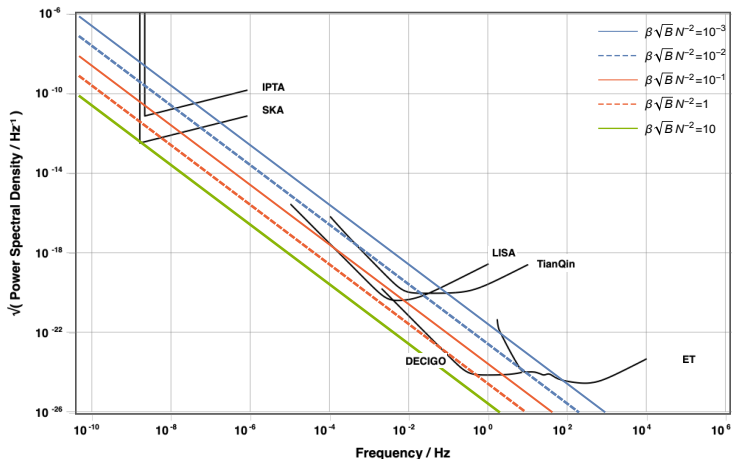
$$N \gg 1$$

$$g^2 = 10^{-36} \quad \lambda = 10^{-72} \quad \boxed{N = 24} \quad B = 1$$



$$A = \sqrt{\frac{3\Omega_{gw,peak} H_0^2}{2\pi^2 f_{peak}^2}}$$

Gravitational waves vs sensitivity curves



Strain $\sqrt{S_h}$

$$\Omega_{gw} H_0^2 = \frac{2\pi^2 f^3}{3} S_h$$

gwplotter.com Moore, Cole, and Berry'14

Chapter 4.

A bit of dark matter
and the return of Zeldovich.

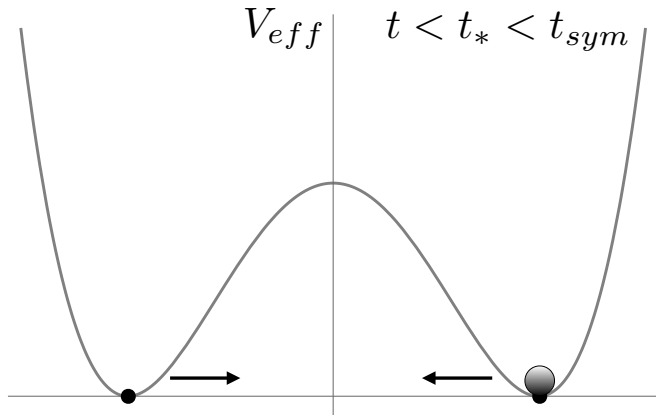
Chapter 4.

A bit of dark matter
and the return of Zeldovich.

Slightly break conformal invariance.

$$\mathcal{L} = \frac{(\partial_\mu \chi)^2}{2} - \frac{M^2 \cdot \chi^2}{2} - \frac{\lambda \cdot \chi^4}{4} + \frac{g^2 \chi^2 \phi^\dagger \phi}{2}.$$

Inverse phase transition and beyond freeze-in Dark Matter



At early times χ tracks the minimum $\chi = v$

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$$v = \sqrt{\frac{Ng^2 T^2}{12\lambda} - \frac{M^2}{\lambda}}$$

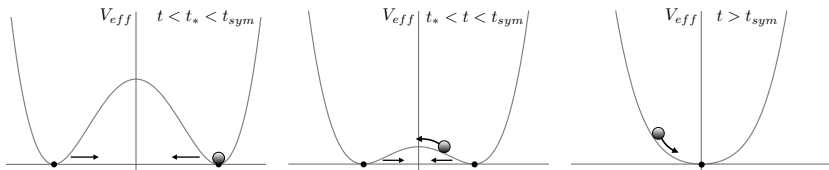
$$\frac{dv}{dt} \propto \frac{1}{v} \rightarrow \infty \text{ as } v \rightarrow 0$$

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χ stops tracking minimum and starts oscillating at low T

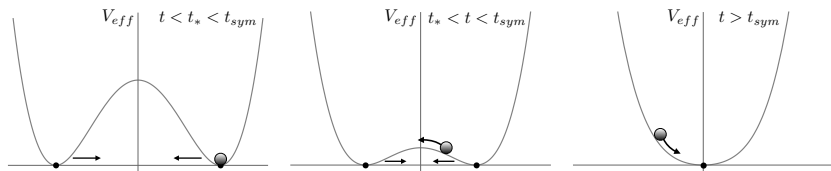


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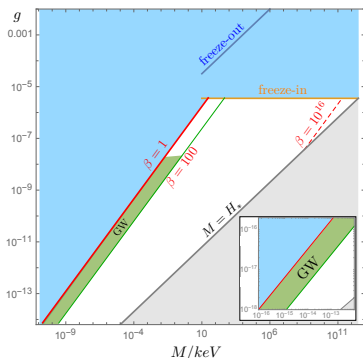
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χ stops tracking minimum and starts oscillating at low T



Z_2 -symmetry + feeble couplings involved protect stability
 \implies these oscillations naturally feed into dark matter

Abundance constraint: $M \simeq 3 \times 10^{-13} \text{ eV} \cdot \frac{\beta^{3/5}}{\sqrt{N}} \cdot \left(\frac{g}{10^{-18}} \right)^{7/5}$



$M \simeq 10^{-12} - 10^{-13} \text{ eV} \implies$ superradiance Zeldovich

Thanks for your attention!!!