Analytic Formulae for Inflationary Correlators with Dynamical Mass

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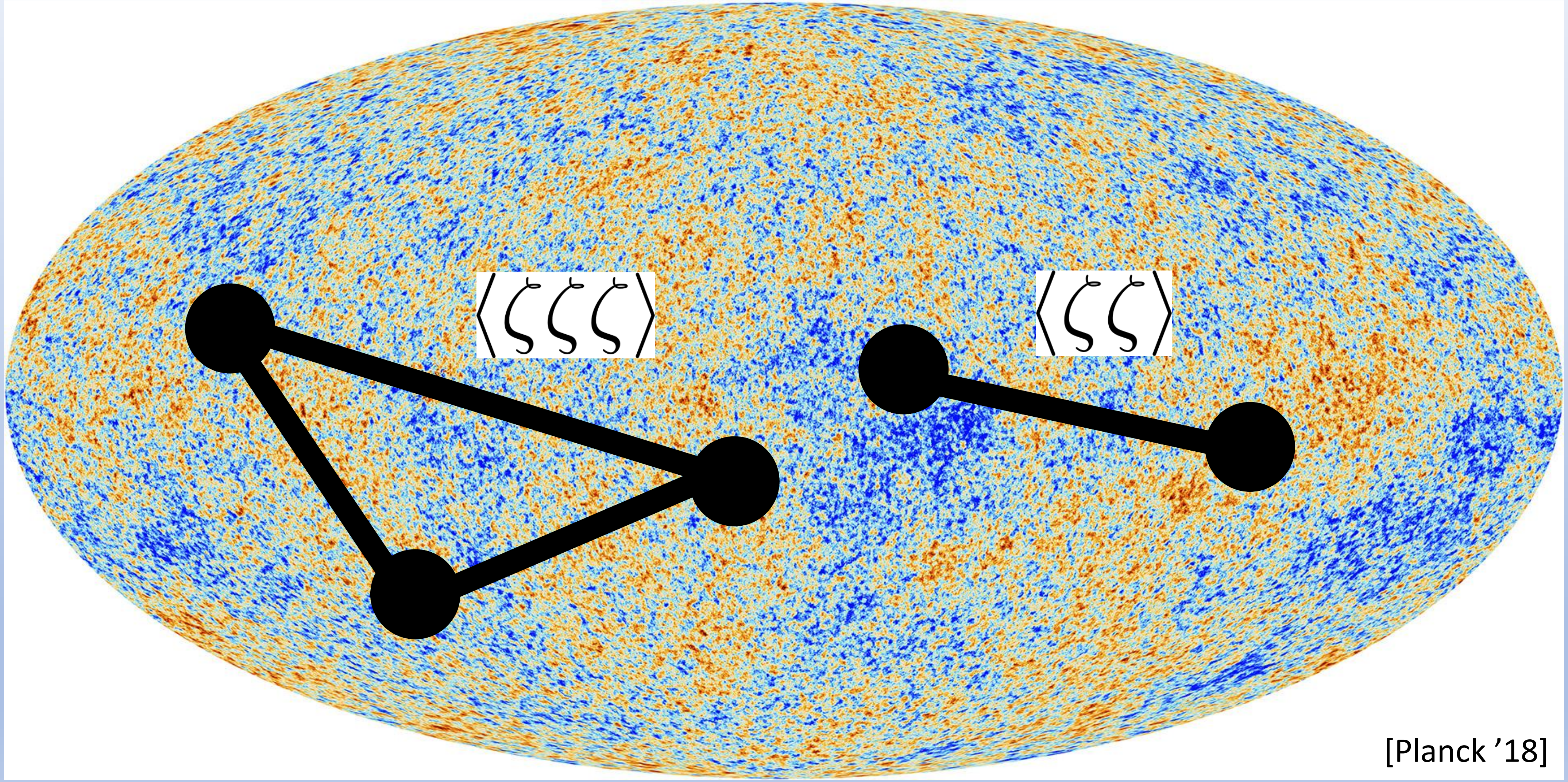
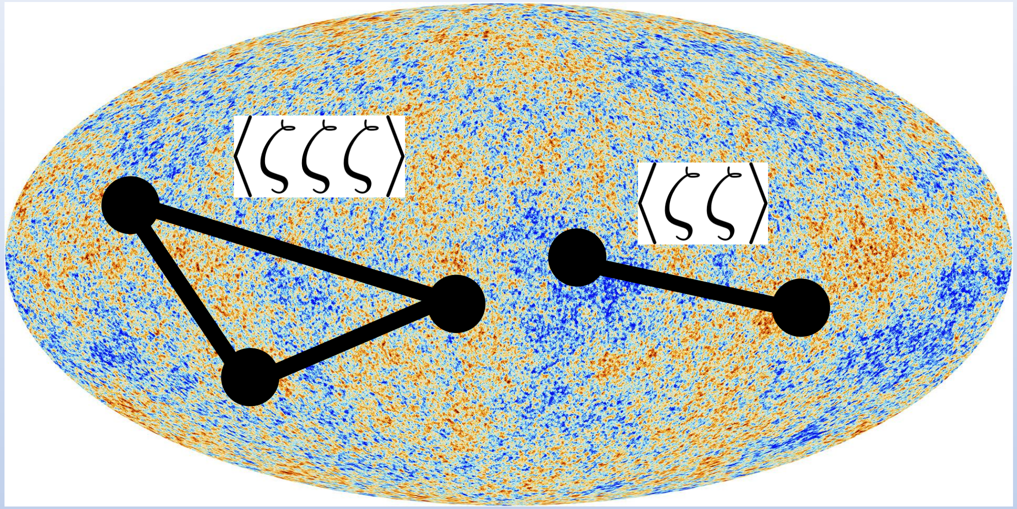
Observables for Inflationary Cosmology



ζ : Scalar curvature perturbation $\langle \zeta \cdots \zeta \rangle$: Correlation functions

Observables for Inflationary Cosmology



2pt. correlation function (power spectrum)

$$\langle \zeta_{\boldsymbol{k}} \zeta_{\boldsymbol{k}'} \rangle_{\text{inf. end}} = (2\pi)^3 \delta^3(\boldsymbol{k} + \boldsymbol{k}') \frac{2\pi^2}{k^3} P_\zeta$$

$$P_\zeta \simeq \frac{H^2}{8\pi^2 \epsilon} \left(\frac{k}{k_*} \right)^{n_s - 1} \quad , \quad n_s \simeq 0.965 \quad \frac{dn_s}{d\log k} \simeq 0.002$$

[Planck '18]

➡ Degeneracy of inflation models

3pt. correlation function (bispectrum)

$$\langle \zeta_1 \zeta_2 \zeta_3 \rangle_{\text{inf. end}} = (2\pi)^7 \delta^3(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3) \frac{P_{\zeta *}^2}{(k_1 k_2 k_3)^2} S\left(\frac{k_1}{k_3}, \frac{k_2}{k_3} \right)$$

[Chen and Wang '09]

S : dim. less, model dependent shape function

$\langle\zeta\zeta\zeta\rangle$: effects of interactions in perturb. theory of QFT

➡ Probe for inflation models and BSM physics

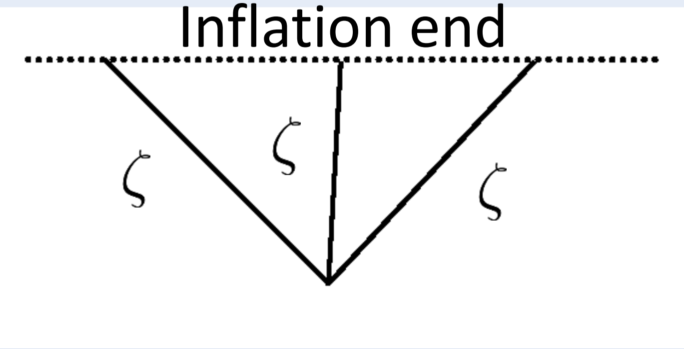
Bispectrum in Minimal Inflation

Leading: contact diagram

Third order interactions of curvature perturb.

$$\mathcal{L}^{(3)} = a^3\epsilon^2\zeta\dot{\zeta}^2 + a\epsilon^2\zeta(\partial\zeta)^2 + \cdots$$

[Maldacena ’03]



Bispectrum: $S \to \frac{k_1}{4k_3}(1-n_s)$ in $k_3 \ll k_1 \simeq k_2$

Observation

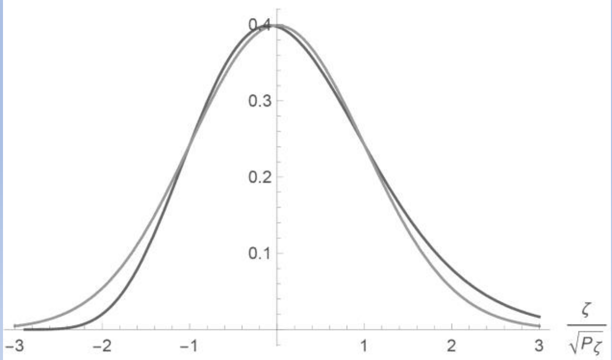
Gaussian distribution

Non-linearity parameter $f_{\rm NL}^{\rm local}$: $\zeta(x) = \zeta_g(x) - \frac{3}{5}f_{\rm NL}^{\rm local}\zeta_g^2(x) + \cdots$

$$\Rightarrow \langle\zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle = (2\pi)^7\delta^3(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)\frac{3f_{\rm NL}^{\rm local}}{10k_1^3k_2^3}P_\zeta^2 + (\text{perm. of } k_1, k_2, k_3)$$

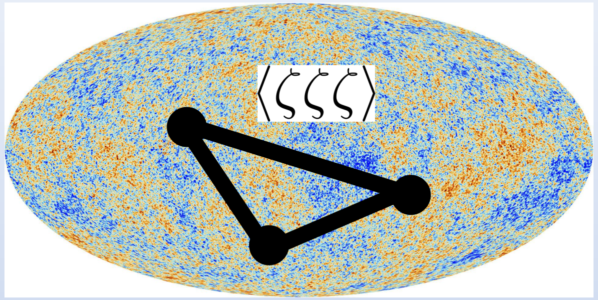
Planck 2018 $f_{\rm NL}^{\rm local} = -0.9 \pm 5.1$

Future resolution: 21 cm line $\mathcal{O}(10^{-2})$



Cosmological Collider Project

[Noumi et al. ’12,
Arkani-Hamed and Maldacena, ’15]



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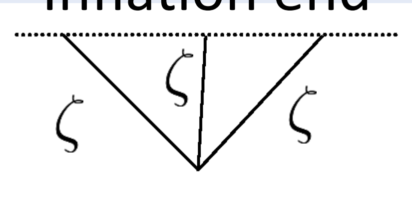
Inflation end



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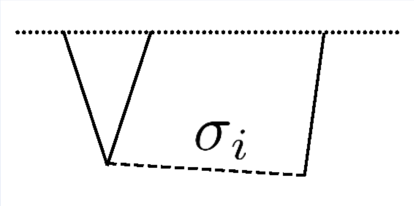

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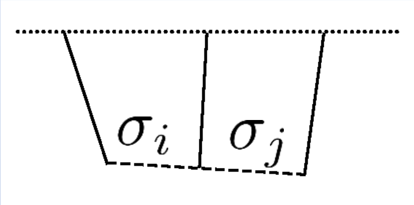

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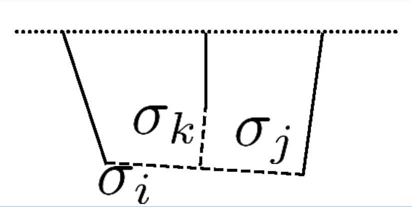


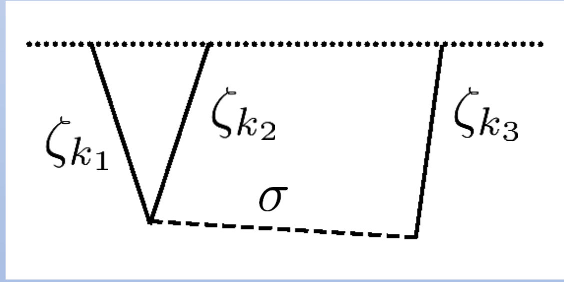
$+ \cdots$

Signals for massive particles [Chen and Wang ’09, Noumi et al. ’12]

$$S \sim \left(\frac{k_3}{k_1}\right)^{1/2} e^{-\pi\mu} \underline{\underline{\cos\left(\mu \log\frac{k_1}{k_3}\right)}}$$

$k_3 \ll k_1 \simeq k_2$

$$\mu = \sqrt{\left(\frac{m_\sigma}{H}\right)^2 - \frac{9}{4}}$$



Mass: wavelength of the shape function

Dictionary for particles of BSM in high energy scale $H \lesssim 10^{13}\ \mathrm{GeV}$

Supersymmetry, gauge symmetry, CP violation, swampland, ...

[Baumann and Green ’12] [Maru and Okawa ’21] [Liu et al. ’21] [Reece et al. ’22]

Q. Distinction of Interactions?

A classification of interactions

1. Derivative ints.: $f(\partial_\mu\phi, \sigma, \partial_\mu\sigma)$

- respect shift sym. of ϕ

(de Sitter spacetime)

- EFT, SUGRA, etc.

2. Non-derivative ints.: $f(\phi, \sigma, \partial_\mu\sigma)$

- break shift sym.

(slow-roll effects)

- Higgs, axion, extra dim., etc.

$\phi\bar{\psi}\psi \quad \phi F^{\mu\nu}F_{\mu\nu} \quad e^{\alpha\phi/M_{\rm pl}}\sigma^2$

Interactions ➡ BSM, inflation models

BUT in previous works: [Noumi et al. '12, etc.]

- no information about types of ints.

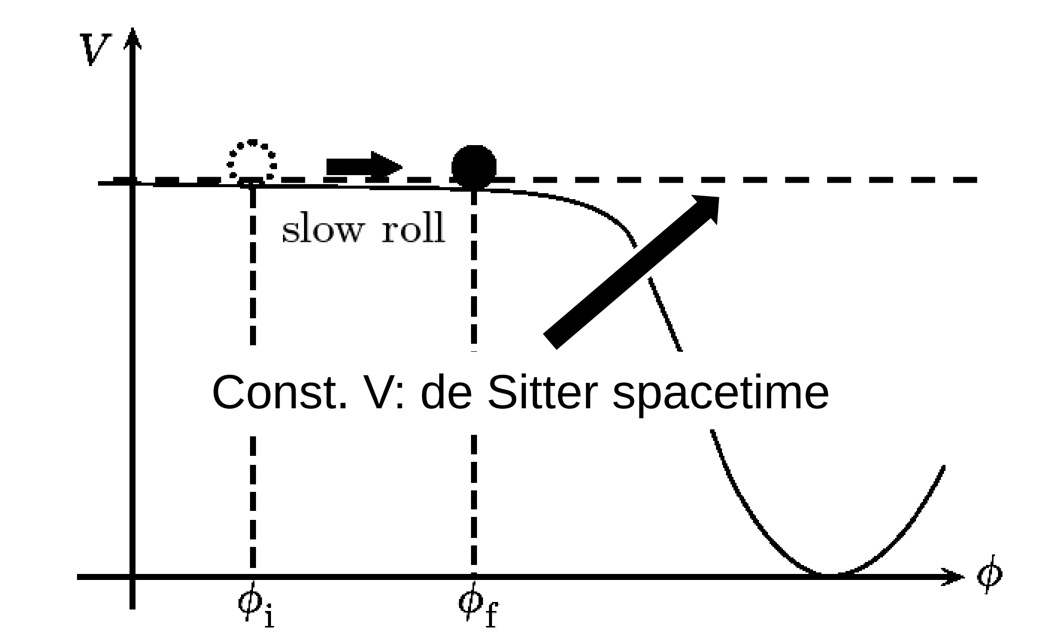
$$S \sim \left(\frac{k_3}{k_1}\right)^{1/2} e^{-\pi\mu} \cos\left(\mu \log \frac{k_1}{k_3}\right)$$

(not in signals of colliders on ground as well?)

- scale invariant approx.

Scale dependence ⬌ shift sym. breaking

➡ Non-der. ints.: large scale dependence?



Demonstration of Scale Dependence (our calculation)

Approximated / numerical results: [Wang '19, Reece et al. '22]

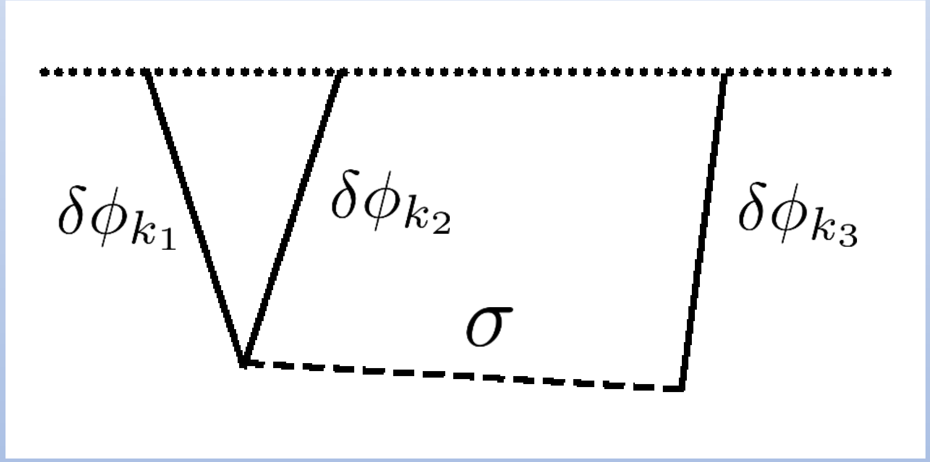
Action for Inflaton ϕ + massive scalar spectator σ

$$S = \int dx^4 \sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R - \frac{1}{2}(\partial_\mu\phi)^2 - V(\phi) - \frac{1}{2}(\partial_\mu\sigma)^2 - \frac{1}{2}m_0^2\sigma^2 - M_{\rm pl}y\phi\sigma^2 + \mathcal{L}_{\rm diag}\right]$$

Sym. breaking interaction

Diagram

$$\mathcal{L}_{\rm diag} \supset c_2(-\tau)^{-3}\sigma\delta\phi' + c_3(-\tau)^{-2}\sigma(\delta\phi')^2$$



Time dependent mass (excursion of inflaton)

$$m_{\rm eff}^2 = m_0^2 + 2yM_{\rm pl}\phi_0$$

$$\phi_0' = \frac{\sqrt{2\epsilon}M_{\rm pl}}{\tau}$$

Slow-roll approx. $\phi_0 = \sqrt{2\epsilon}M_{\rm pl}\log\frac{\tau}{\tau_0}$

Linear approx. $\phi_0(\tau) \simeq \phi_{*0} - \sqrt{2\epsilon}M_{\rm pl}\left(1 - \frac{\tau}{\tau_*}\right)$

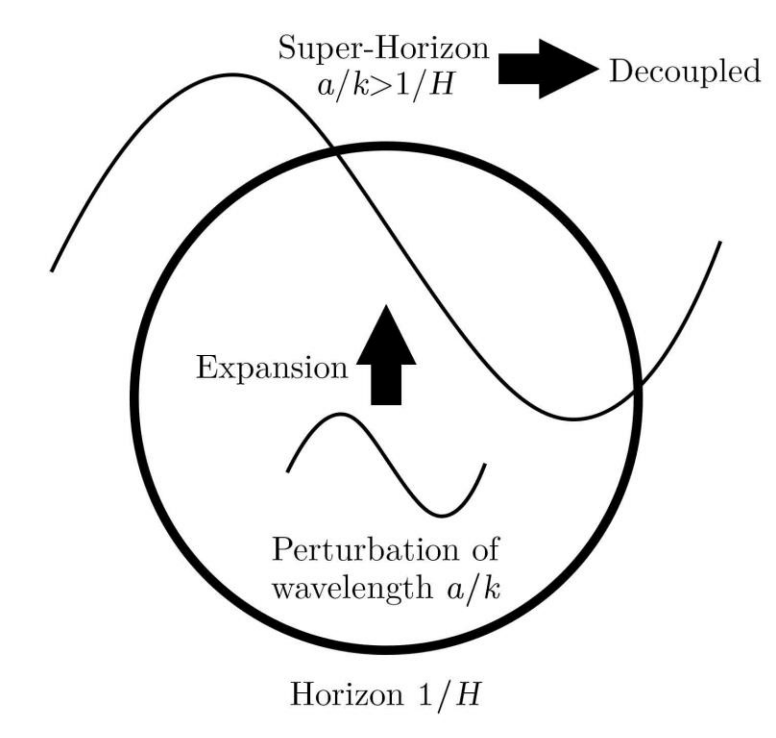
Initial condition $\phi_{*0} \simeq \sqrt{2\epsilon}M_{\rm pl}\log\frac{\tau_*}{\tau_0}$

Additional scale τ_0, τ_*

Effects of Time Dependent Mass

Evo. of perturb. ⬅ **Time of horizon crossing** ⬌ **Scales**

$k\tau = -1$



Constant mass: Scale invariant $S(k_1/k_3, k_2/k_3)$

Time dependent mass: Scale dependent

Different mass for each horizon crossing scale

➡ Dependence of absolute values of scales

Fixing additional scales

Two additional scales τ_0, τ_*

$$\begin{cases} k_i \tau_* = -1 \quad \text{(expansion at horizon crossing)} \\ \phi_{*0} \simeq \sqrt{2\epsilon} M_{\rm pl} \log\left(\frac{k_0}{k_*}\right) \sim \mathcal{O}\left(N_{\rm CMB}\sqrt{2\epsilon}M_{\rm pl}\right) \end{cases}$$ at the largest scale of CMB

τ_0, τ_* ⬌ k_0, k_*

$k_* \sim 10^{-4}\ {\rm Mpc}^{-1}$

Comments on Mass Correction

Inflaton mass $\mathcal{O}(\epsilon)H^2$ vs. mass correction to massive scalar

Non-derivative coupling $\mathcal{L}_{\rm mass} = -\frac{1}{2}g(\phi)\sigma^2$

$$\mu^2 = \frac{g_* - g_{\phi,*}\sqrt{2\epsilon}M_{\rm pl}}{H^2} - \frac{9}{4}$$: effective mass of isocurvaton

E.g., $g = m_0^2 + \frac{\alpha}{\Lambda^{n-2}}\phi^n$

$$\left|\alpha\left(\frac{M_{\rm pl}}{\Lambda}\right)^{n-2}\left(\frac{M_{\rm pl}}{H}\right)^2\epsilon^{n/2}\right| \gg \mathcal{O}(\epsilon,\eta)$$ ➡ $n \lesssim 10$ for $\Lambda = M_{\rm pl}, \alpha \sim \mathcal{O}(1)$

Derivative coupling $\left(m_0^2 + \frac{\beta}{\Lambda^{m(n+1)-2}}(\phi^{(n)})^m\right)\sigma^2$ **:**

nm: even

$$\left|\beta\left(\frac{H}{\Lambda}\right)^{nm}\left(\frac{M_{\rm pl}}{\Lambda}\right)^{n-2}\left(\frac{M_{\rm pl}}{H}\right)^2\epsilon^{nm-n/2}\right| \gg \mathcal{O}(\epsilon,\eta)$$ ➡ Impossible to be valid

$\Lambda = M_{\rm pl}, \beta \sim \mathcal{O}(1)$

Mode Functions of the Heavy Field

Mode expansion

$$\sigma(x) = \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\left(v_k(\tau)a_{\boldsymbol{k}} + v_k^*(\tau)a^\dagger_{-\boldsymbol{k}}\right)e^{i\boldsymbol{k}\cdot\boldsymbol{x}} ,$$

$$[a_{\boldsymbol{k}}, a^\dagger_{\boldsymbol{k}'}] = (2\pi)^3\delta(\boldsymbol{k}-\boldsymbol{k}')$$

v_k : Mode function

Equation of motion for σ

$$v_k'' - \frac{2}{\tau}v_k' + \left(k^2 + \frac{m_{\rm eff}^2}{H^2\tau^2}\right)v_k = 0 ,$$

$$m_{\rm eff}^2 = m_0^2 + 2yM_{\rm pl}^2\left[\frac{\phi_{*0}}{M_{\rm pl}} \mp \sqrt{2\epsilon}\left(1 - \frac{\tau}{\tau_*}\right)\right]$$

Mode functions for σ (Bunch-Davies vacuum)

$$v_k = \frac{e^{\pi\beta/4k}}{\sqrt{2k}}(-H\tau)W_{-i\beta/2k, i\mu}(2ik\tau)$$

$$\mu^2 = \frac{1}{H^2}\left(m_0^2 + 2yM_{\rm pl}\phi_{*0} \mp 2y\sqrt{2\epsilon}M_{\rm pl}^2\right) - \frac{9}{4}$$

$$\beta = \pm\frac{2y\sqrt{2\epsilon}M_{\rm pl}^2}{\tau_* H^2}$$

$y \to 0$: const. mass mode function

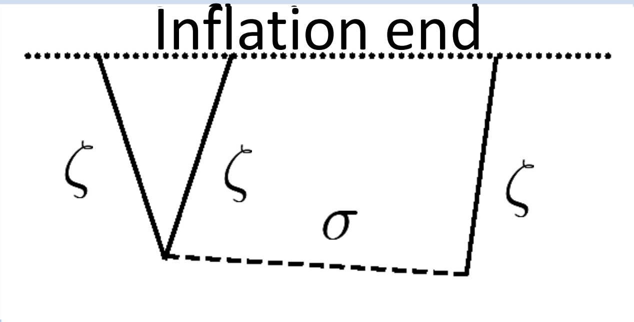
$$v_k = e^{-\pi\mu/2}\frac{\sqrt{\pi}}{2}H(-\tau)^{3/2}H_{i\mu}^{(1)}(-k\tau)$$

Analytic Calculation of CC signals

How to calculate the correlation function precisely?

Massive mode function: $v_k = e^{-\pi\mu/2}\frac{\sqrt{\pi}}{2}H(-\tau)^{3/2}H^{(1)}_{i\mu}(-k\tau)$ $\quad \mu^2 = \frac{m^2}{H^2} - \frac{9}{4}$

Time-ordered integration for vertices… $\int_{-\infty}^{0} dx_2 \int_{-\infty}^{x_2} dx_1 x_1^n x_2^m e^{i(x_1 - x_2)} H^{(1)}_{i\mu}(x_1) H^{(2)}_{-i\mu}(x_2)$



Quantitatively?

Previous approaches:
- Numerical integration
- Super-horizon approx. $k\tau \to 0$

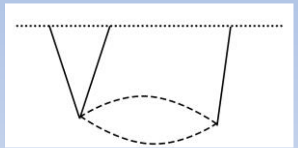
Reliability?

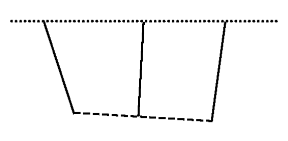
Newly developed approaches: Bootstrapping and Mellin-Barnes representation

Basic idea → Shown later in our work

Single exchange of scalar & vector, one-loop of scalar, double exchange of scalar,…

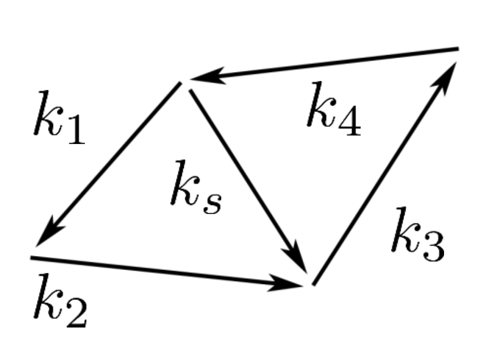
[Qin and Xianyu ’22 and ’23] [Xianyu and Zhang ’22] [Xianyu and Zang ’23]

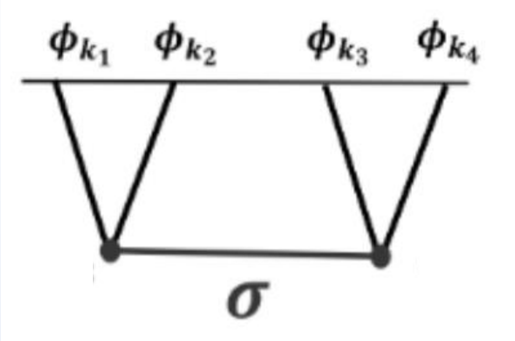




Bootstrap method

[Arkani-Hamed ’18, Pimentel ’22, Jazayeri ’22, Qin ’22 etc.]





Seed integrals

$$\mathcal{I}_{\mathrm{ab}}^{p_1 p_2} = -\mathrm{ab} k_s^{5+p_{12}} \int_{-\infty}^{0} \mathrm{d}\tau_1 \; \mathrm{d}\tau_2 \, (-\tau_1)^{p_1} \, (-\tau_2)^{p_2} \, e^{\mathrm{i}\mathrm{a}k_{12}\tau_1 + \mathrm{i}\mathrm{b}k_{34}\tau_2} D_{\mathrm{ab}}\left(k_s; \tau_1, \tau_2\right)$$

Bispectrum $\langle \zeta^3 \rangle \propto \frac{c_2 c_3}{8 k_1 k_2 k_3^4} \lim_{k_4 \to 0} \sum_{\mathrm{a,b}=\pm} \mathcal{I}_{\mathrm{ab}}^{0,-2} + (k_3 \to k_1, k_2)$ (In-in formalism)

Equation of motion for propagator $D_{\mathrm{ab}}\left(k_s; \tau_1, \tau_2\right)$, and $\partial_\tau \leftrightarrow \partial_k$ thanks to combination $k\tau$

Bootstrap equations for seed integrals

$$\mathcal{D}_{\pm, u_i}^{p_1} \mathcal{I}_{\pm\mp}^{p_1 p_2} = 0 \,, \quad \mathcal{D}_{\pm, u_i}^{p_1} \mathcal{I}_{\pm\pm}^{p_1 p_2} = H^2 e^{\mp \mathrm{i} p_{12}\pi/2} \Gamma\left(5 + p_{12}\right) \left(\frac{u_1 u_2}{2\left(u_1 + u_2 - u_1 u_2\right)} \right)^{5+p_{12}}$$

$$\mathcal{D}_{\pm, u}^{p} \equiv \left(u^2 - u^3\right) \partial_u^2 - \left[(4+2p)u - (1 + p \pm \mathrm{i}\gamma) u^2\right] \partial_u + \left[\mu^2 + \left(p + \frac{5}{2}\right)^2\right]$$

New term

$$u_1 = \frac{2k_s}{k_{12} + k_s} \qquad u_2 = \frac{2k_s}{k_{34} + k_s} \qquad \gamma = \pm \frac{y\sqrt{2\epsilon} M_{\mathrm{pl}}^2}{H^2} \frac{k}{k_0}$$

Integration of special functions → Ordinary differential eq.

Boundary Conditions

Solutions for seed integrals

General solutions of seed integrals

$$\mathcal{I}_{ab} = \sum_{c,d=\pm} A_{ab|cd}\mathcal{V}_{a|c}(u_1)\mathcal{V}_{b|d}(u_2) + \mathcal{G}_{ab}(u_1, u_2)$$

$A_{ab|cd}$: integration constant

Boundary conditions fixing $A_{ab|cd}$

Direct integration of correlators using Mellin-Barnes representation

$$W_{\kappa,\nu}(z) = e^{z/2}\int_{-i\infty}^{i\infty}\frac{ds}{2\pi i}\frac{\Gamma(s-\nu)\Gamma(s+\nu)}{\Gamma(s-\kappa+1/2)}z^{-s+1/2}$$

Integration of special function ➡ Infinite sum of residues

In the limit $k_s \to 0$, the summations are resolved

➡ The coefficients at the limit should coincide with $A_{ab|cd}$

$$A_{ab|++} = A_{ab|+-} = \frac{\pm iH^2 e^{\pi(\kappa+\mu)}\cosh(\pi(\kappa-\mu))}{\pi\Gamma(1/2-i\mu\mp i\kappa)\Gamma(1/2+i\mu\mp i\kappa)}e^{\mp i\pi(p_1+p_2)/2} \qquad A_{ab|-+} = A_{ab|--} : \ \mu \to -\mu$$

Analytical Results

□ : scale dependence
□ : same as const. mass signal

Bispectrum

$$S = \sum_{a,b=\pm}\left[\frac{k_1k_2}{k_3^2}\mathcal{U}_{ab}^{0,-2}\left(\frac{2k_3}{k_{123}},\frac{k_3}{k_0}\right)+\frac{k_2k_3}{k_1^2}\mathcal{U}_{ab}^{0,-2}\left(\frac{2k_1}{k_{123}},\frac{k_1}{k_0}\right)+\frac{k_3k_1}{k_2^2}\mathcal{U}_{ab}^{0,-2}\left(\frac{2k_2}{k_{123}},\frac{k_2}{k_0}\right)\right]$$

where

$$\mathcal{U}_{\pm\pm}^{p_1p_2}(u,v) = D_1(p_1,p_2,\mu_v,\gamma_v)u^{5+p_{12}}\,{}_3F_2\left[\begin{matrix}1,3+p_2\mp i\gamma_v,5+p_{12}\\ \frac{7}{2}+p_2-i\mu_v,\frac{7}{2}+p_2+i\mu_v\end{matrix}\middle|\,u\right]$$
$$\mp D_2(p_1,p_2,\mu_v,\gamma_v)u^{5/2+p_1\pm i\mu_v}F\left[\begin{matrix}p_1+\frac{5}{2}\pm i\mu_v,\frac{1}{2}\pm i\mu_v\mp i\gamma_v\\ 1\pm 2i\mu_v\end{matrix}\middle|\,u\right]+(\mu_v\to-\mu_v)$$

$$\mathcal{U}_{\pm\mp}^{p_1p_2}(u,v) = C(p_1,p_2,\mu_v,\gamma_v)u^{5/2+p_1\pm i\mu_v}F\left[\begin{matrix}p_1+\frac{5}{2}\pm i\mu_v,\frac{1}{2}\pm i\mu_v\mp i\gamma_v\\ 1\pm 2i\mu_v\end{matrix}\middle|\,u\right]+(\mu_v\to-\mu_v)$$

$$k_{123}=k_1+k_2+k_3\ ,\qquad \gamma_v=\pm\frac{y\sqrt{2\epsilon}M_{\rm pl}^2}{H^2}v\ ,\qquad \mu_v^2=\frac{1}{H^2}\left(m_0^2+2y\sqrt{2\epsilon}M_{\rm pl}^2\log v\mp 2y\sqrt{2\epsilon}M_{\rm pl}^2\right)-\frac{9}{4}$$

from mode function

from evaluation at horizon crossing

Observational Signals

cf. const. mass

$$S \sim \left(\frac{k_3}{k_1}\right)^{1/2} e^{-\pi\mu} \cos\left(\mu \log \frac{k_3}{k_1}\right) \qquad \mu = \sqrt{\left(\frac{m_0}{H}\right)^2 - \frac{9}{4}}$$

Scale dependence : mass of short mode at the time of horizon crossing

$$S \sim \left(\frac{k_3}{k_1}\right)^{1/2} e^{-\pi\mu\left(v\frac{k_3}{k_1}\right)} \cos\left[\mu\left(v\frac{k_3}{k_1}\right) \log \frac{k_3}{k_1}\right] \quad \text{in} \quad k_3 \ll k_1 \simeq k_2$$

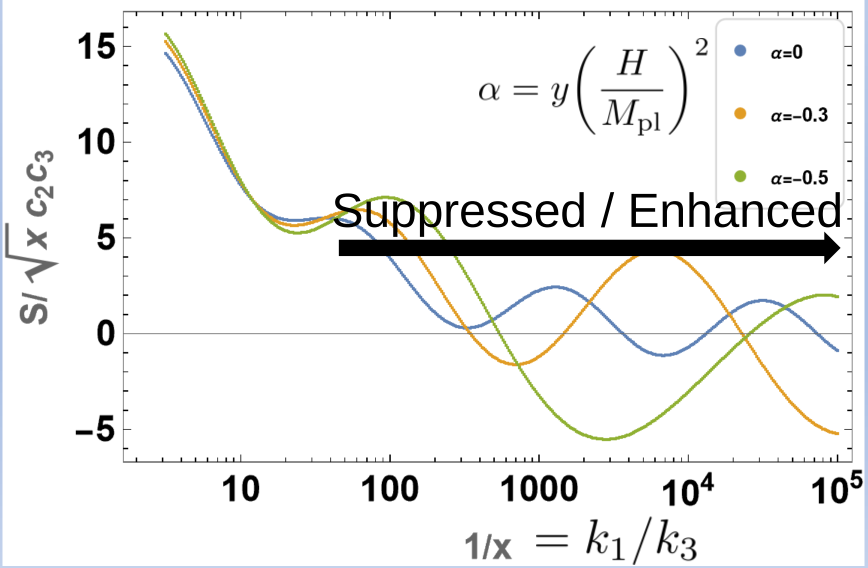
$$v = k_1/k_0 = 10^4 k_1, \quad \mu^2 = \frac{1}{H^2}\left(m_0^2 + 2y\sqrt{2\epsilon}M_{\rm pl}^2 \log\left(v\frac{k_3}{k_1}\right) \mp 2y\sqrt{2\epsilon}M_{\rm pl}^2\right) - \frac{9}{4}$$

$$\Delta\phi \sim N\sqrt{\epsilon}M_{\rm pl}$$

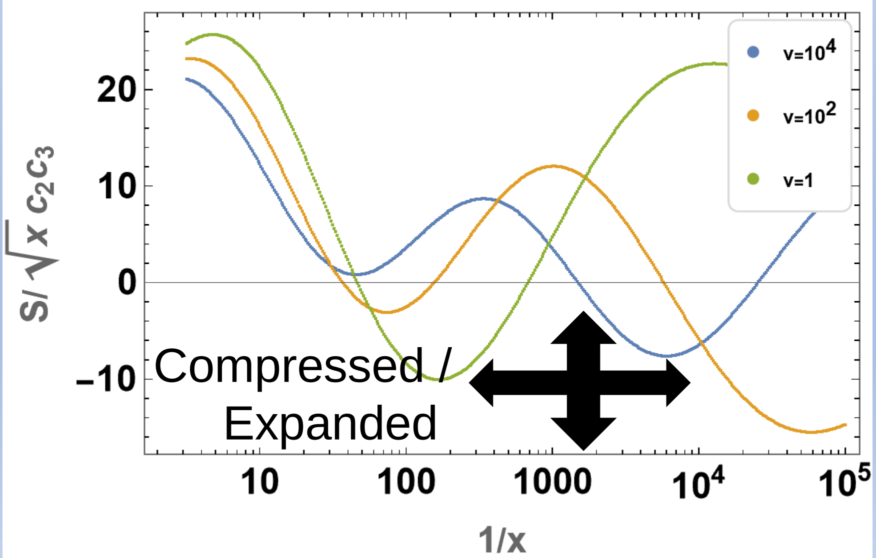
[Lyth '96]

Not slow-roll suppressed thanks to the hierarchy $M_{\rm pl}/H \gtrsim 10^5$ in case of non-der. ints.

Coupling dependence



Scale dependence



Probing Ints. 1: Der. vs Non-Der. Ints.

Scale dependence : mass at horizon crossing

(1) Non-derivative coupling $\frac{\alpha}{\Lambda^{n-2}}\phi^n\sigma^2$

$$\frac{\Delta m_{\rm eff}^2}{H^2} \simeq \alpha\left(\frac{M_{\rm pl}}{\Lambda}\right)^{n-2}\left(\frac{M_{\rm pl}}{H}\right)^2 \epsilon^{n/2}\left(\log\left(v\frac{k_3}{k_1}\right)\right)^n$$

➡ Large scale dependence

(2) Derivative coupling $\frac{\beta}{\Lambda^{n(m+1)-2}}(\partial^m\phi)^n\sigma^2$ $\quad nm$: even

$$\frac{\Delta m_{\rm eff}^2}{H^2} \simeq \beta\left(\frac{H}{\Lambda}\right)^{nm}\left(\frac{M_{\rm pl}}{\Lambda}\right)^{n-2}\left(\frac{M_{\rm pl}}{H}\right)^2 \epsilon^{nm-n/2}\left(\log\left(v\frac{k_3}{k_1}\right)\right)^n$$

➡ Stronger suppression ⬅

(same order as the signal)

➡ Large scale dependence $\Leftrightarrow$ Non-derivative coupling

Probing Ints. 2: Among Non-Der. Ints.

Boltzmann suppression of the signal $S \sim \left(\frac{k_3}{k_1}\right)^{1/2} \underline{\underline{e^{-\pi\mu}}} \cos\left(\mu \log \frac{k_3}{k_1}\right)$

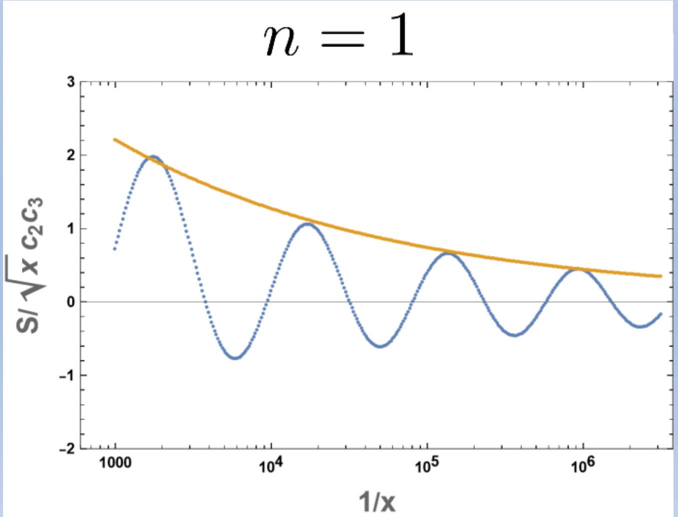
E.g., power function $\frac{\alpha}{\Lambda^{n-2}}\phi^n\sigma^2$ ➡ $\frac{\Delta m_{\rm eff}^2}{H^2} \simeq \alpha \left(\frac{M_{\rm pl}}{\Lambda}\right)^{n-2} \left(\frac{M_{\rm pl}}{H}\right)^2 \epsilon^{n/2} \left(\log\left(v\frac{k_3}{k_1}\right)\right)^n$

$$\Rightarrow \quad e^{-\pi\mu} \sim \exp\left[-\pi\sqrt{\alpha\left(\frac{M_{\rm pl}}{\Lambda}\right)^{n-2}\left(\frac{M_{\rm pl}}{H}\right)^2 \epsilon^{n/2} \underline{\underline{\left(\log\left(v\frac{k_3}{k_1}\right)\right)^n}}}\right]$$

Determination of n from the suppression

More generally, $\mathcal{L}_{\rm int} = g(\phi)\sigma^2$

$$\Rightarrow \quad e^{-\pi\mu} \sim \exp\left[-\frac{\pi}{H}\sqrt{\underline{\underline{g\left(M_{\rm pl}\sqrt{2\epsilon}\log\left(v\frac{k_3}{k_1}\right)\right)}}}\right]$$



Suppression rate is
uniquely characterized by $g(\phi)$

Summary

BSM

$$\frac{\alpha}{M_{\rm pl}^{n-2}}\phi^n\sigma^2$$

$$\frac{\beta}{M_{\rm pl}^{n(m+1)-2}}(\partial^m\phi)^n\sigma^2$$

⋮

Cosmological collider project: $k_3 \ll k_1 \simeq k_2$

- Dictionary for particles $S \sim \left(\frac{k_3}{k_1}\right)^{1/2} e^{-\pi\mu}\cos\left(\mu\log\frac{k_3}{k_1}\right)$
- Scale dependence: the type of interactions

Signals: horizon crossing (e.g., $\mu \to \mu(vk_3/k_1)$)

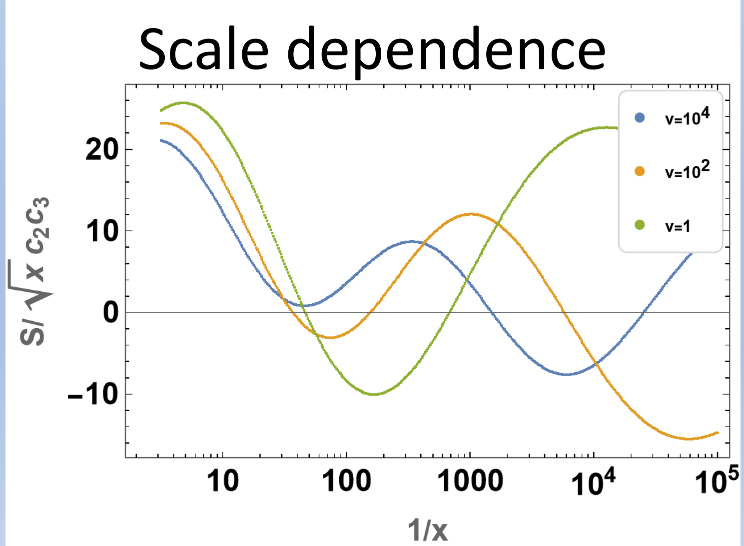
Distinguishing ints. by scale dependence in $\Delta m_{\rm eff}$:

O Derivative vs. Non-derivative interactions

Derivative ints. Non-derivative ints.

$$\left(\frac{H}{M_{\rm pl}}\right)^{nm-2}\epsilon^{nm-n/2}\left(\log\left(v\frac{k_3}{k_1}\right)\right)^n << \left(\frac{M_{\rm pl}}{H}\right)^2\epsilon^{n/2}\left(\log\left(v\frac{k_3}{k_1}\right)\right)^n$$

Observably large thanks to $M_{\rm pl}/H \gtrsim 10^5$



O Determining a non-der int. $g(\phi)\sigma^2$

$$e^{-\pi\mu} \sim \exp\left[-\frac{\pi}{H}\sqrt{g\left(M_{\rm pl}\sqrt{2\epsilon}\log\left(v\frac{k_3}{k_1}\right)\right)}\right]$$

Appendices

Interpretation of CC Signal as Interference

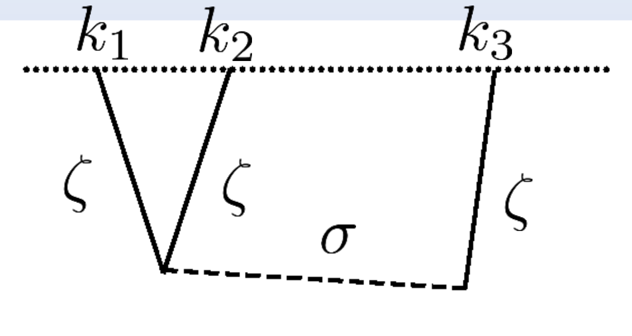
The oscillatory signal is generically produced as shown below

$$S \sim \left(\frac{k_3}{k_1}\right)^{1/2} e^{-\pi\mu} \cos\left(\mu \log \frac{k_1}{k_3}\right)$$

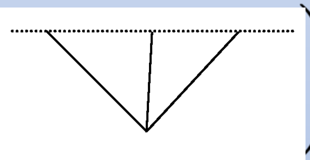
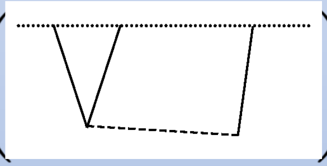
$k_3 \ll k_1 \simeq k_2$

$$\mu = \sqrt{\left(\frac{m_\sigma}{H}\right)^2 - \frac{9}{4}}$$

[Chen and Wang ’09, Noumi et al. ’12]



Observation: quantum superposition

$\Bigg|\Bigg($  $\Bigg)^{1/2} + \Bigg($  $\Bigg)^{1/2}\Bigg|^2$ ⬅ Cross term: interference

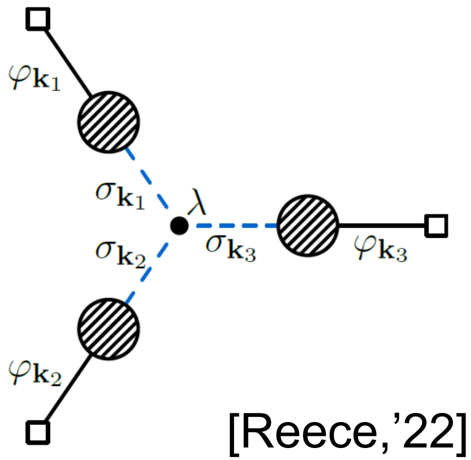
Oscillatory signal $\left(\frac{k_1}{k_3}\right)^{i\mu} \sim \left(\frac{\tau_3}{\tau_1}\right)^{i\mu} \sim e^{im(t_1 - t_3)}$: interference factor

Boltzmann suppression

Hawking temperature $\beta = \frac{2\pi}{H}$ ➡ Particle prod. $\langle\sigma\sigma\rangle \sim e^{-2\pi\mu} \sim e^{-(2\pi/H)m}$

➡ The interference part is a dominant oscillatory signal ➡ $e^{-\pi\mu}$

Non-derivative Ints., Numerical [Reece '22]



[Reece,'22]

Setup

Effective mass of the heavy particle: $m_{\rm eff} = e^{\alpha\phi/M_{\rm pl}} m_0^2$

Oscillatory feature

Wavelength } amplified or dumped
Amplitude

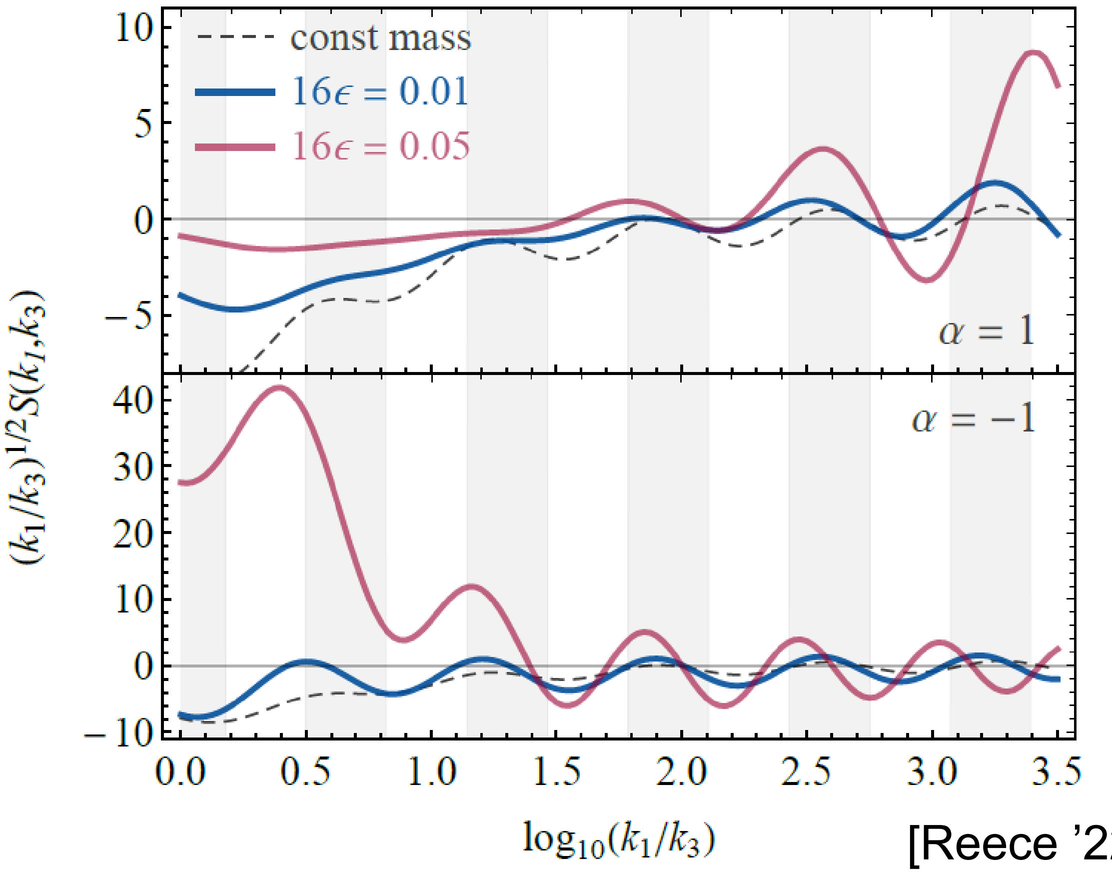
Things not clear in numerical work:

-physical interpretation

-model dependence

-scale dependence

Breaking down of de Sitter



[Reece '22]

Equilateral limit $k_1 = k_2 = k_3$

$v = k_1/k_0$ dependence

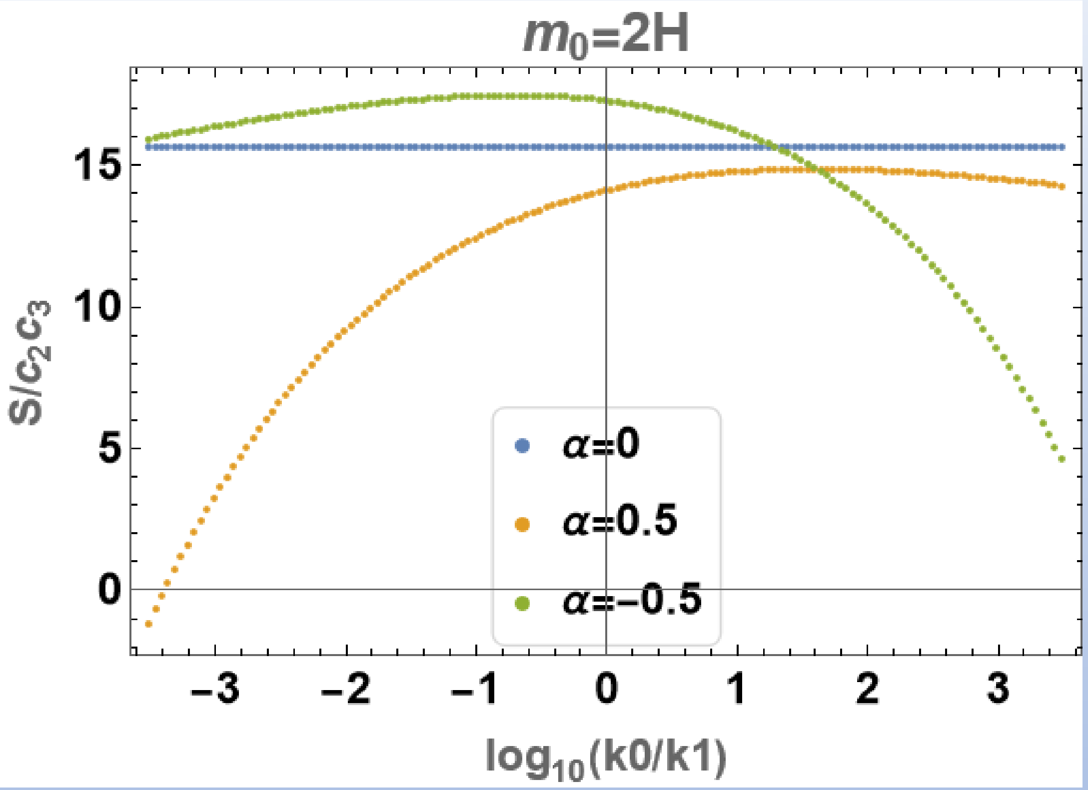
$$\frac{\partial S}{\partial v} = f(m_0)\frac{\sqrt{\epsilon}\alpha}{v} + \mathcal{O}(\epsilon)$$

The same scale dependence as

the general single field inflation [Chen, '07]

(Consistent to EFT description

integrating out heavy field)

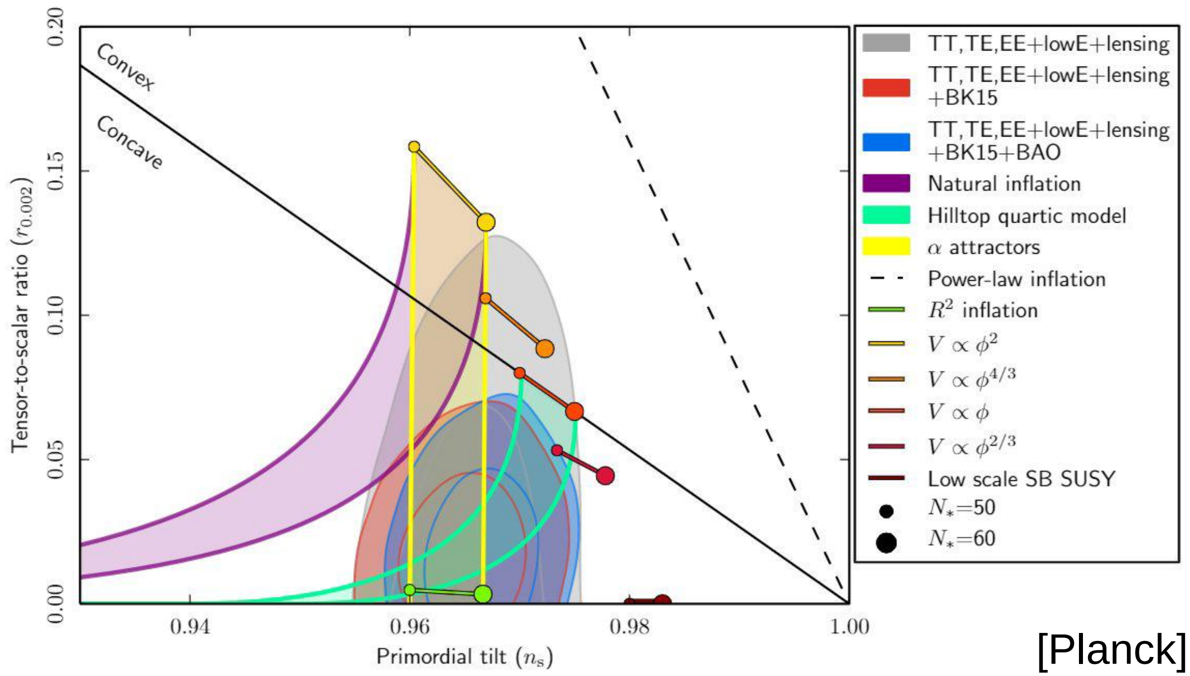


Amplitude

$$S_{\rm eq}(\approx f_{\rm NL}^{\rm eq}) \sim c_2 c_3 \mathcal{O}(10)$$

$c_2 c_3$: dim. less ➡ $\mathcal{O}(1)$? $\mathcal{O}(\epsilon)$?

Planck 2018



[Planck]

Linear perturbations:

$$P_\zeta \simeq 2 \times 10^{-9}, \quad n_s \simeq 0.0965$$

Tensor: not yet detected

$$r = \frac{P_\gamma}{P_\zeta} < 0.056$$

Isocurvature perturbation: not detected

➡ Single field inflation is preferred.

Non-Gaussianities:

Squeezed: $f_{\rm NL}^{\rm local} = -0.9 \pm 5.1$, Equilateral: $f_{\rm NL}^{\rm equil} = -26 + 47$

Form factor: insufficient resolution

Future experiment: 21 cm line ➡ resolution $\mathcal{O}(10^{-2})$