Constructing Symmetric Teleparallel theories with the Gauss-Bonnet invariant

Leonardo G. Trombetta
CEICO - Czech Academy of Sciences - Prague

based on:

Juan Manuel Armaleo, Sebastian Bahamonde, Georg Trenkler, L.G.T.
arXiv: 2308.07299 - PRD (in press)

Funded by the European Union

MARIE CURIE ACTIONS

FZU Fyzikální ústav Akademie věd České republiky

ceico

CAS-JSPS-IBS CTPU-CGA Autumn Workshop on Cosmology, Gravity, Particle Physics

Prague, October 10th, 2023

Outline

- Motivations
- Introduction
  - Theories with the Gauss-Bonnet invariant
  - Teleparallel gravity
- Gauss-Bonnet invariant in General Teleparallel formulation
  - Bulk and boundary splits
  - Flat FLRW
- Example: scalar-Symmetric-Teleparallel-Gauss-Bonnet theory

Why modified gravity?

- Account for the “invisible” ingredients: Dark Energy, Dark Matter
- H_0 and σ_8 tensions
- Singularities
- Quantum gravity
- Strong gravity regime needs to be tested
- A good way to understand GR is to modify it

General Relativity - Assumptions

General Relativity is based upon different assumptions that can be understood as the fulfilling of the Lovelock's theorem. Some assumptions are:

- **Equivalence principle**
- **General covariance:** Invariant under diffeomorphisms and Local Lorentz transformations.
- **Riemannian geometry:** The connection is the Levi-Civita one.
- **4-dimensions**
- **2nd order derivatives:** gravitational action contains only second derivatives.
- **Locality**

How to modify it?

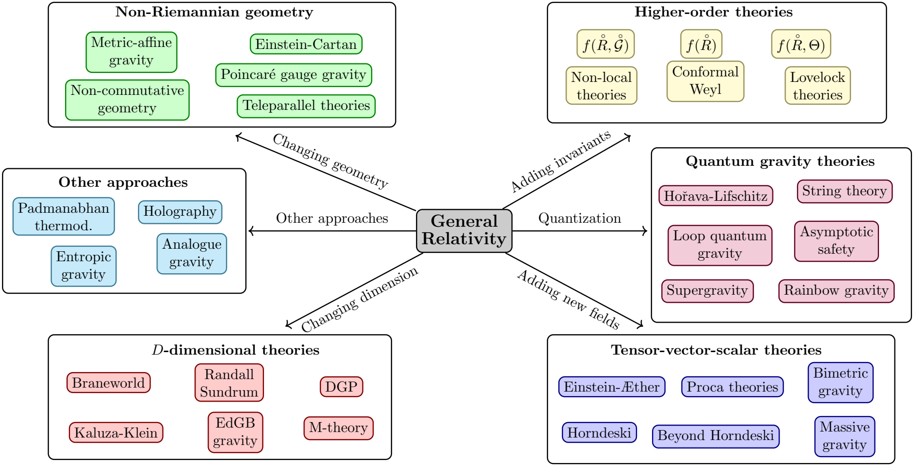


Figure: Classification of theories of gravity. (S. Bahamonde et.al., [arXiv:2106.13793 [gr-qc]].)

Gauss-Bonnet invariant

$$G = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2,$$

- Quadratic in the curvature
- $D > 4$:

$$S = \int d^D x \sqrt{-g}\, G$$

  Leads to second-order field equations

- $D = 4$: Topological invariant $\Longrightarrow$ Pure boundary term $\Longrightarrow$ No change in the dynamics

$$G = \nabla_\mu \left[\Gamma^3\right]^\mu$$

  Divergence of a non-diff-invariant vector

Theories with Gauss-Bonnet in $D = 4$

How to make the Gauss-Bonnet invariant nontrivial in $D = 4$:

- $f(G)$ theories

$$S = \int d^4x \sqrt{-g}\, f(G)$$

  No longer second order

- scalar-Gauss-Bonnet

$$S = \int d^4x \sqrt{-g}\, f(\phi)\, G$$

  belongs to Horndeski (Kobayashi, Yamaguchi, Yokoyama (2011))
  $\Longrightarrow$ Second-order field equations

Scalar-Gauss-Bonnet: linear coupling

Sotiriou and Zhou (2014)

$$\mathcal{L} = M_P^2 R - \frac{1}{2}(\partial\phi)^2 + M_P\alpha\,\phi G$$

- Shift-symmetric
- Horndeski ($G_5 \sim \log X$)
- Evades no-hair theorems by Hui & Nicolis (2012)

Scalar EOM around Schwarzschild

$$\Box\phi = M_P\alpha\, G = \frac{12 M_P\alpha\, r_s^2}{r^6}$$

Black holes

$$\phi \simeq \frac{Q_s}{r} \qquad \text{(long-range)}$$

with

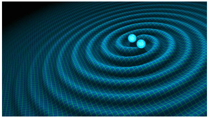
$$Q_s = \frac{M_P\alpha}{r_s}$$

Other sources

$$\phi \simeq \frac{M_P\alpha r_s^2}{r^4} \qquad \text{(short-range)}$$

Bounds

Observational constraint: Inspiral



Dephasing due to scalar wave emission:

$$\alpha \lesssim (1.2\text{km})^2$$

Lyu, Jiang, Yagi (2022)

Deviations from GR $< 1\%$ for black holes of $10M_\odot$

Causality constraint:

Absence of superluminal propagation:

$$\Lambda_{UV} < \frac{1}{\sqrt{\alpha}} \sim 1\,\text{km}^{-1}$$

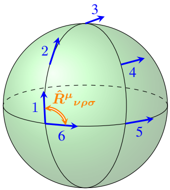
Serra, Serra, Trincherini, LGT (2022)

Teleparallel gravity

Geometric notions

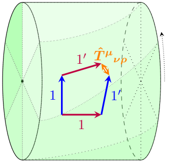
Curvature tensor $R^{\alpha}{}_{\beta\mu\nu}$

Rotation experienced by a vector when it is parallel transported along a closed curve



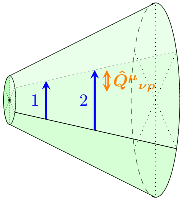
Torsion tensor $T^{\alpha}{}_{\mu\nu}$

non-closure of the parallelogram formed when two infinitesimal vectors are parallel transported along each other.



Non-metricity tensor $Q_{\alpha\mu\nu}$

measures how much the length and angle of vectors change as we parallel transport them, so in metric spaces the length of vectors is conserved



Fundamental variables and characteristic tensors

- In the most general metric-affine setting, the fundamental variables are a **metric** $g_{\mu\nu}$ (10 comp.) as well as the coefficients $\Gamma^{\rho}{}_{\mu\nu}$ (64 comp.) of an **affine connection**.
- The most general connection can be written as

Connection decomposition

$$\Gamma^{\lambda}{}_{\mu\nu} = \underbrace{\mathring{\Gamma}^{\lambda}{}_{\mu\nu}}_{\text{Levi-Civita}} + \underbrace{\frac{1}{2}\,T^{\lambda}{}_{\mu\nu} - T_{(\mu}{}^{\lambda}{}_{\nu)}}_{\text{Torsion part: } K^{\lambda}{}_{\mu\nu}} + \underbrace{\frac{1}{2}\,Q^{\lambda}{}_{\mu\nu} - Q_{(\mu}{}^{\lambda}{}_{\nu)}}_{\text{Nonmetricity part: } L^{\lambda}{}_{\mu\nu}} = \mathring{\Gamma}^{\lambda}{}_{\mu\nu} + N^{\lambda}{}_{\mu\nu}$$

where we have defined the following geometrical tensors use

Curvature	$R^{\mu}{}_{\nu\rho\sigma} = \partial_{\rho}\Gamma^{\mu}{}_{\nu\sigma} - \partial_{\sigma}\Gamma^{\mu}{}_{\nu\rho} + \Gamma^{\mu}{}_{\tau\rho}\Gamma^{\tau}{}_{\nu\sigma} - \Gamma^{\mu}{}_{\tau\sigma}\Gamma^{\tau}{}_{\nu\rho}$
Torsion	$T^{\mu}{}_{\nu\rho} = \Gamma^{\mu}{}_{\rho\nu} - \Gamma^{\mu}{}_{\nu\rho}$
Nonmetricity	$Q_{\mu\nu\rho} = \nabla_{\mu} g_{\nu\rho} = \partial_{\mu} g_{\nu\rho} - \Gamma^{\sigma}{}_{\nu\mu} g_{\sigma\rho} - \Gamma^{\sigma}{}_{\rho\mu} g_{\nu\sigma}$

Curvature tensor decomposition

- The curvature becomes

$$R^{\mu}{}_{\nu\rho\sigma} = \mathring{R}^{\mu}{}_{\nu\rho\sigma} + \mathring{\nabla}_{\rho} N^{\mu}{}_{\nu\sigma} - \mathring{\nabla}_{\sigma} N^{\mu}{}_{\nu\rho} + N^{\mu}{}_{\tau\rho} N^{\tau}{}_{\nu\sigma} - N^{\mu}{}_{\tau\sigma} N^{\tau}{}_{\nu\rho} \,.$$

- Contracting the curvature tensor to obtain the Ricci scalar $R = g^{\mu\nu} R^{\rho}{}_{\mu\rho\nu}$ we find

Ricci scalar decomposition

$$R = \mathring{R} + \left(T + 2\mathring{\nabla}_{\mu}(\sqrt{-g} T^{\rho}{}_{\rho}{}^{\mu}) \right) + \left(Q + \mathring{\nabla}_{\mu} Q^{\mu\nu}{}_{\nu} - \mathring{\nabla}_{\nu} Q_{\mu}{}^{\mu\nu} \right) + C$$

with

$$T := T^{\rho\lambda\kappa} T_{\rho\lambda\kappa} + 2\, T^{\rho\lambda\kappa} T_{\kappa\rho\lambda} - 4\, T_{\rho}{}^{\kappa}{}_{\kappa} T^{\rho\lambda}{}_{\lambda} \,, \quad \text{Torsion scalar}\,,$$

$$Q := -\frac{1}{4}\, Q_{\alpha\beta\gamma} Q^{\alpha\beta\gamma} + \frac{1}{2}\, Q_{\alpha\beta\gamma} Q^{\beta\alpha\gamma} + \frac{1}{4}\, Q_{\alpha} Q^{\alpha} - \frac{1}{2}\, Q_{\alpha} \bar{Q}^{\alpha} \,, \text{ Nonmetricity scalar}\,,$$

$$C := 2(Q_{\kappa\rho\lambda} T^{\lambda\kappa\rho} + Q_{\rho}{}^{\sigma}{}_{\sigma} T^{\rho\kappa}{}_{\kappa} - Q^{\sigma}{}_{\sigma\rho} T^{\rho\kappa}{}_{\kappa}) \,.$$

Teleparallel geometries

- **General Teleparallel geometry** $(R_{\alpha\mu\nu\beta} = 0)$: In the case of vanishing curvature, the connection is flat.

- **Torsional Teleparallel geometry** $(R_{\alpha\mu\nu\beta} = 0, Q_{\alpha\mu\nu} = 0)$: The metric satisfies the metric-compatibility condition but torsion is non-zero.

- **Symmetric Teleparallel geometry** $(R_{\alpha\mu\nu\beta} = 0, T^{\alpha}{}_{\mu\nu} = 0)$: Both torsion tensor and curvature are zero and the gravitational interactions are only mediated through non-metricity.

Trinity of gravity

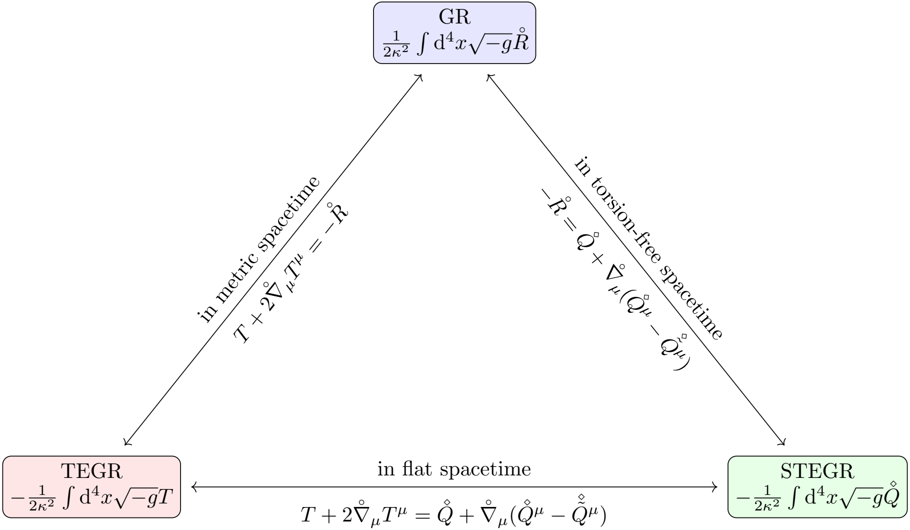


Figure: Geometrical trinity of gravity (S. Bahamonde et.al., [arXiv:2106.13793 [gr-qc]].)

Extending GR

- Modifications of GR differ depending of the geometrical formulation.

$$\mathring{R} = T + B_T = Q + B_Q$$

And then:

$$f(\mathring{R}) \neq f(T) \neq f(Q)$$

We can no longer drop the total derivative terms B_T or B_Q.

- What about Gauss-Bonnet?

In the case of Torsional Teleparallel geometry it has been shown:

Kofinas, Saridakis (2014)

$$\mathring{G} = T_G^{(T)} + B_G^{(T)}$$

Then we can construct theories like:

$$S = \int \mathrm{d}^4x \sqrt{-g} f(T, B_T, T_G^{(T)}, B_G^{(T)})$$

or by coupling $T_G^{(T)}$ and $B_G^{(T)}$ with a scalar independently.

General teleparallel Gauss-Bonnet invariant

Riemannian Gauss-Bonnet invariant:

$$\overset{\circ}{G} = \overset{\circ}{R}_{\mu\nu\rho\sigma}\overset{\circ}{R}^{\mu\nu\rho\sigma} - 4\overset{\circ}{R}_{\mu\nu}\overset{\circ}{R}^{\mu\nu} + \overset{\circ}{R}^2$$

In differential form language (in D dimensions):

$$\overset{\circ}{G} *1 = \frac{1}{(D-4)!}\epsilon_{a_1\cdots a_D}\overset{\circ}{R}^{a_1 a_2} \wedge \overset{\circ}{R}^{a_3 a_4} \wedge e^{a_5} \wedge \cdots \wedge e^{a_D}$$

Under the teleparallel condition $R^{ab} = 0$

$$\overset{\circ}{R}^{a_1 a_2} = -\overset{\circ}{\mathrm{D}} N^{a_1 a_2} - N^{a_1}{}_f \wedge N^{f a_2}$$

$$\begin{aligned}\overset{\circ}{G} *1 \quad = \quad & \frac{1}{(D-4)!}\epsilon_{a_1\cdots a_D}\Big[-\mathrm{d}\left(N^{a_1 a_2} \wedge \overset{\circ}{R}^{a_3 a_4} \wedge e^{a_5} \wedge \cdots \wedge e^{a_D}\right) \\ & - N^{a_1}{}_f \wedge N^{f a_2} \wedge \overset{\circ}{R}^{a_3 a_4} \wedge e^{a_5} \wedge \cdots \wedge e^{a_D}\Big]\end{aligned}$$

We now replace $\overset{\circ}{R}^{a_3 a_4}$ once more time in the second term,

$$\begin{aligned}\overset{\circ}{G} *1 \quad = \quad & \frac{1}{(D-4)!}\epsilon_{a_1\cdots a_D}\Big[-\mathrm{d}\left(N^{a_1 a_2} \wedge \overset{\circ}{R}^{a_3 a_4} \wedge e^{a_5} \wedge \cdots \wedge e^{a_D}\right) \\ & + \left(N^{a_1}{}_f \wedge N^{f a_2} \wedge \overset{\circ}{\mathrm{D}} N^{a_3 a_4} + N^{a_1}{}_f \wedge N^{f a_2} \wedge N^{a_3}{}_h \wedge N^{h a_4}\right) \wedge e^{a_5} \wedge \cdots \wedge e^{a_D}\Big]\end{aligned}$$

Gauss-Bonnet invariant - tensorial form

$$\overset{\circ}{G} = {}^2T_G^{(T,Q)} + {}^2B_G^{(T,Q)}$$

$$ {}^2T_G^{(T,Q)} \quad = \quad \delta^{\mu\,\nu\,\rho\,\sigma}_{\mu_1\mu_2\mu_3\mu_4}\left[N^{\mu_1}{}_{\alpha\mu}N^{\alpha\mu_2}{}_{\nu}N^{\mu_3}{}_{\beta\rho}N^{\beta\mu_4}{}_{\sigma} - N^{\mu_1}{}_{\alpha\mu}N^{\alpha\mu_2}{}_{\nu}\overset{\circ}{\nabla}_{\sigma}N^{\mu_3\mu_4}{}_{\rho}\right]$$

$$ {}^2B_G^{(T,Q)} \quad = \quad -\frac{1}{2}\frac{1}{\sqrt{-g}}\,\partial_\mu\left[\sqrt{-g}\,\delta^{\mu\,\nu\,\rho\,\sigma}_{\mu_1\mu_2\mu_3\mu_4}N^{\mu_1\mu_2}{}_{\nu}\overset{\circ}{R}{}^{\mu_3\mu_4}{}_{\rho\sigma}\right]$$

where

$$\delta^{\mu\,\nu\,\rho\,\sigma}_{\mu_1\mu_2\mu_3\mu_4} = \frac{1}{(D-4)!}\epsilon_{\mu_1\ldots\mu_D}\epsilon^{\mu\nu\rho\sigma\mu_5\ldots\mu_D}$$

Gauss-Bonnet invariant - alternative split

$$\overset{\circ}{G} = {}^{1}\,T_G^{(T,Q)} + {}^{1}B_G^{(T,Q)}$$

$$\begin{aligned}
{}^{1}\,T_G^{(T,Q)} &= \delta^{\mu\,\nu\,\rho\,\sigma}_{\mu_1\mu_2\mu_3\mu_4}\Big[N^{\mu_1}{}_{\alpha\mu}N^{\alpha\mu_2}{}_{\nu}N^{\mu_3}{}_{\beta\rho}N^{\beta\mu_4}{}_{\sigma} - 2N^{\mu_1\mu_2}{}_{\mu}N^{\mu_3}{}_{\alpha\nu}N^{\alpha}{}_{\beta\rho}N^{[\beta\mu_4]}{}_{\sigma} \\
&\quad +2g_{\alpha\beta}N^{\mu_1\mu_2}{}_{\mu}N^{[\mu_3\alpha]}{}_{\nu}N^{\beta\mu_4}{}_{\gamma}N^{\gamma}{}_{\rho\sigma} + 2g_{\alpha\beta}N^{\mu_1\mu_2}{}_{\mu}N^{[\mu_3\alpha]}{}_{\nu}\nabla_{\sigma}N^{\beta\mu_4}{}_{\rho} \\
&\quad +4g_{\alpha\beta}g_{\gamma\delta}N^{\mu_1\mu_2}{}_{\mu}N^{[\mu_3\alpha]}{}_{\nu}N^{\mu_4\gamma}{}_{\rho}N^{(\delta\beta)}{}_{\sigma}\Big]
\end{aligned}$$

$${}^{1}B_G^{(T,Q)} = \frac{1}{\sqrt{-g}}\,\partial_\mu\left[\sqrt{-g}\,\delta^{\mu\,\nu\,\rho\,\sigma}_{\mu_1\mu_2\mu_3\mu_4}N^{\mu_1\mu_2}{}_{\nu}\left(N^{\mu_3}{}_{\lambda\rho}N^{\lambda\mu_4}{}_{\sigma} - \frac{1}{2}\overset{\circ}{R}{}^{\mu_3\mu_4}{}_{\rho\sigma}\right)\right]$$

Properties

- Known Torsional teleparallel expression is recovered for $N^{\lambda}{}_{\mu\nu} \to K^{\lambda}{}_{\mu\nu}$ ($Q^{\lambda}{}_{\mu\nu} \to 0$) in the first set

- In $D = 4$:

$$ {}^{i}T_{G}^{(T,Q)} = \mathring{G} - {}^{i}B_{G}^{(T,Q)} = \text{boundary term} $$

- All pieces ${}^{i}T_{G}^{(T,Q)}$ and ${}^{i}B_{G}^{(T,Q)}$ contain up to second order derivatives of $g_{\mu\nu}$ and $\Gamma^{\lambda}{}_{\mu\nu}$

Symmetric teleparallel conditions

Adak et al. (2006), Beltrán Jiménez et al. (2017)

- No torsion:

$$T^{\lambda}{}_{\mu\nu} = \Gamma^{\lambda}{}_{\mu\nu} - \Gamma^{\lambda}{}_{\nu\mu} = 0 \quad \Longrightarrow \quad \Gamma^{\lambda}{}_{\mu\nu} = (\Lambda^{-1})^{\lambda}{}_{\alpha}\partial_{\mu}\Lambda^{\alpha}{}_{\nu}$$

- No curvature:

$$R^{\mu}{}_{\nu\rho\sigma} = \partial_{\rho}\Gamma^{\mu}{}_{\nu\sigma} - \partial_{\sigma}\Gamma^{\mu}{}_{\nu\rho} + \Gamma^{\mu}{}_{\tau\rho}\Gamma^{\tau}{}_{\nu\sigma} - \Gamma^{\mu}{}_{\tau\sigma}\Gamma^{\tau}{}_{\nu\rho} = 0 \quad \Longrightarrow \quad \Gamma^{\lambda}{}_{\mu\nu} = \frac{\partial x^{\lambda}}{\partial \xi^{\alpha}}\partial_{\mu}\partial_{\nu}\xi^{\lambda}$$

- Maximum # of d.o.f. associated to the sym. teleparallel connection = 4

- Coincident gauge:

$$\xi^{\mu} = x^{\mu} \quad \Longrightarrow \quad \Gamma^{\lambda}{}_{\mu\nu} = 0 \quad \Longrightarrow \quad \nabla_{\mu} = \partial_{\mu}$$

Einstein-Hilbert action becomes "Einstein action"; diff-invariance lost

$$S_{\mathsf{STEGR}}|_{\Gamma=0} = \frac{M_{\rm pl}^2}{2}\int d^4x\sqrt{-g}g^{\mu\nu}\left(\mathring{\Gamma}^{\alpha}{}_{\beta\mu}\mathring{\Gamma}^{\beta}{}_{\nu\alpha} - \mathring{\Gamma}^{\alpha}{}_{\beta\alpha}\mathring{\Gamma}^{\beta}{}_{\mu\nu}\right)$$

Symmetric TG Gauss-Bonnet scalars in FRLW

- Consider flat FLRW background
- Ensure the scalar, metric and connection respect the same symmetries: Killing vector fields Z_ζ where $\zeta = \{1..m\}$. Solve

$$\mathcal{L}_{Z_\zeta}\phi = 0 \qquad (\mathcal{L}_{Z_\zeta} g)_{\mu\nu} = 0 \qquad (\mathcal{L}_{Z_\zeta}\Gamma)^{\lambda}{}_{\mu\nu} = 0$$

$$\Longrightarrow \quad ds^2 = -N(t)^2 dt^2 + a(t)^2(dr^2 + r^2 d\vartheta^2 + r^2 \sin^2\vartheta d\varphi^2)$$

For the connection part, it is convenient to decompose the metric as

$$g_{\mu\nu} = -n_\mu n_\nu + h_{\mu\nu}\,, \quad n_\mu = (-N, 0, 0, 0)$$

$$\Longrightarrow \quad Q_{\rho\mu\nu} = 2F_1(t)\, n_\rho n_\mu n_\nu + 2F_2(t)\, n_\rho h_{\mu\nu} + 2F_3(t)\, h_{\rho(\mu} n_{\nu)}$$

- Impose the teleparallel condition (flat connection) $\Longrightarrow$ 3 distinct branches

Branch 1: $F_1 = K\,, F_2 = -H,\ F_3 = 0$
Branch 2: $F_1 = 2H + \frac{1}{KN}\frac{dK}{dt},\ F_2 = -H\,, F_3 = K$
Branch 3: $F_1 = -K - \frac{1}{K}\frac{dK}{dt}\,, F_2 = K - H\,, F_3 = K$

Symmetric TG Gauss-Bonnet scalars in FRLW

$$\overset{\circ}{G} = {}^{1}T_G^{(Q)} + {}^{1}B_G^{(Q)} = {}^{2}T_G^{(Q)} + {}^{2}B_G^{(Q)} = 24H^2\left(\frac{\dot{H}}{N} + H^2\right) .$$

First set

$${}^{1}T_G^{(Q)} = \begin{cases} 24H^2\left(\frac{\dot{H}}{N} + H^2\right) , & \text{First branch} \\ \frac{6(2H+K)^2\dot{H}}{N} + \frac{12H(H+K)\dot{K}}{N} + 6H^2\left(6HK + 4H^2 + 3K^2\right) , & \text{Second branch} \\ \frac{6(K-2H)^2\dot{H}}{N} + \frac{12H(K-H)\dot{K}}{N} + 6H^2\left(-6HK + 4H^2 + 3K^2\right) , & \text{Third branch} \end{cases}$$

$${}^{1}B_G^{(Q)} = \begin{cases} 0 , & \text{First branch} \\ -\frac{6K(4H+K)\dot{H}}{N} - \frac{12H(H+K)\dot{K}}{N} - 18H^2K(2H+K) , & \text{Second branch} \\ \frac{6K(4H-K)\dot{H}}{N} + \frac{12H(H-K)\dot{K}}{N} + 18H^2K(2H-K) , & \text{Third branch} \end{cases}$$

Second set

$${}^{2}T_G^{(Q)} = \begin{cases} -12H^2\left(\frac{\dot{H}}{N} + H^2\right) , & \text{First branch} \\ -\frac{12H(H+K)\dot{H}}{N} - \frac{6H^2\dot{K}}{N} - 6H^3(2H+3K) , & \text{Second branch} \\ \frac{12H(K-H)\dot{H}}{N} + \frac{6H^2\dot{K}}{N} + 6H^3(3K-2H) , & \text{Third branch} \end{cases}$$

$${}^{2}B_G^{(Q)} = \begin{cases} 36H^2\left(\frac{\dot{H}}{N} + H^2\right) , & \text{First branch} \\ \frac{12H(3H+K)\dot{H}}{N} + \frac{6H^2\dot{K}}{N} + 18H^3(2H+K) , & \text{Second branch} \\ \frac{6H(6H-2K)\dot{H}}{N} - \frac{6H^2\dot{K}}{N} + 18H^3(2H-K) . & \text{Third branch} \end{cases}$$

Symmetric TG scalar-Gauss-Bonnet

$$S = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{-g}\Big[Q - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \alpha_1\mathcal{G}_1(\phi)\,T_G^{(Q)} + \alpha_2\mathcal{G}_2(\phi)B_G^{(Q)}\Big]\,.$$

- Riemannian scalar-Gauss-Bonnet is contained as a special case:

$$\alpha_1\mathcal{G}_1(\phi) = \alpha_2\mathcal{G}_2(\phi)$$

- Second order field equations in all the fields $(\phi, g_{\mu\nu}, \Gamma^{\alpha}{}_{\mu\nu})$ for *both* sets

- For the second set *only*, the theory can be written with up to quadratic contractions of $Q_{\lambda\mu\nu} \implies$ It belongs to the Symmetric Teleparallel Horndeski class

Bahamonde, Trenkler, LGT, Yamaguchi (2022)

Summary

- Theories including the Gauss-Bonnet invariant have interesting phenomenology
- Alternative geometrical formulations of gravity provide new ways to modify GR
- We extended the Gauss-Bonnet invariant to General Teleparallel constructions, in particular the Symmetric Teleparallel case.
- We found that the splitting between a bulk and a boundary term is not unique, and in $D = 4$ both pieces are boundary terms.
- In a flat FLRW background in Symmetric Teleparallel formulation 3 branches of solutions exist. The corresponding ST-Gauss-Bonnet invariants include the d.o.f. of the connection $K(t)$ notrivially.
- Example theory: Scalar-ST-Gauss-Bonnet theory has second-order field equations.
- Future directions include:
  - Cosmological evolution
  - Scalarized BHs in scalar-ST-Gauss-Bonnet
  - Generalization to Metric-Affine Gravity
  - ...

Thank you for your attention!