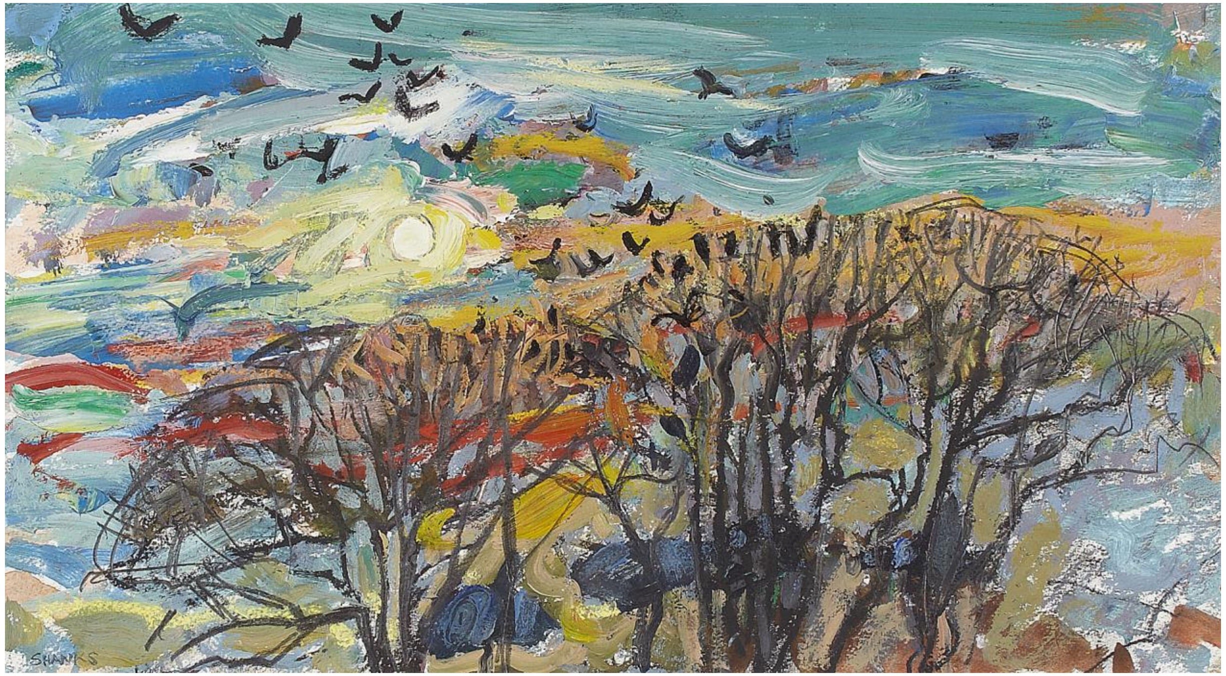
Gravity and parameterized field theories

Tom Zlosnik

University of Gdańsk



Summary of talk:

- Introduction to parameterized field theories (PFTs) as generally covariant versions of non-gravitational theories.

- General Relativity via gauging of global symmetries of PFTs:

  - Einstein-Cartan gravity
  - Macdowell-Mansouri gravity

- Extensions to General Relativity via gauging of subset of global symmetries of PTFs.

- Conclusions

Symmetries:

$$S[\phi^A] = \int_M d^N x L(\eta, \phi^A)$$

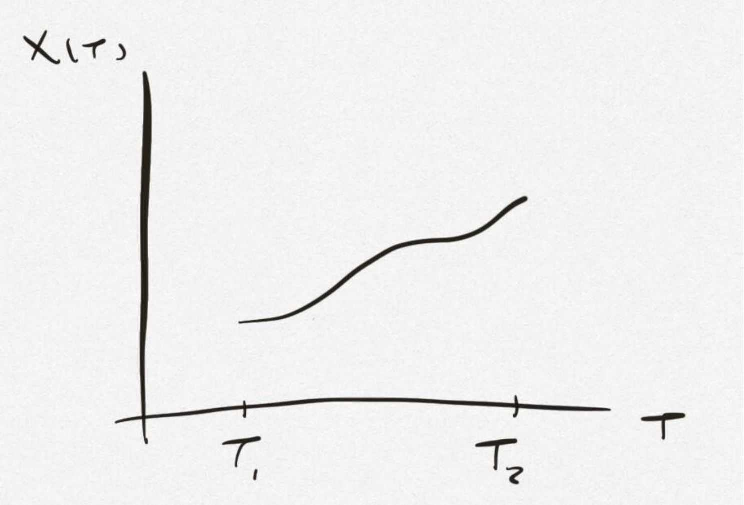
$$\phi^A \to \phi^A + \epsilon^i \chi_i^A(\eta, \phi, \partial\phi, \dots)$$

$$\delta_\epsilon S = 0$$ (Internal symmetries)

$$\delta_\epsilon S = \int_{\partial M} d^{N-1} x \partial_\mu (\xi^\mu(\epsilon) L)$$ (Spacetime symmetries: diffeomorphisms)

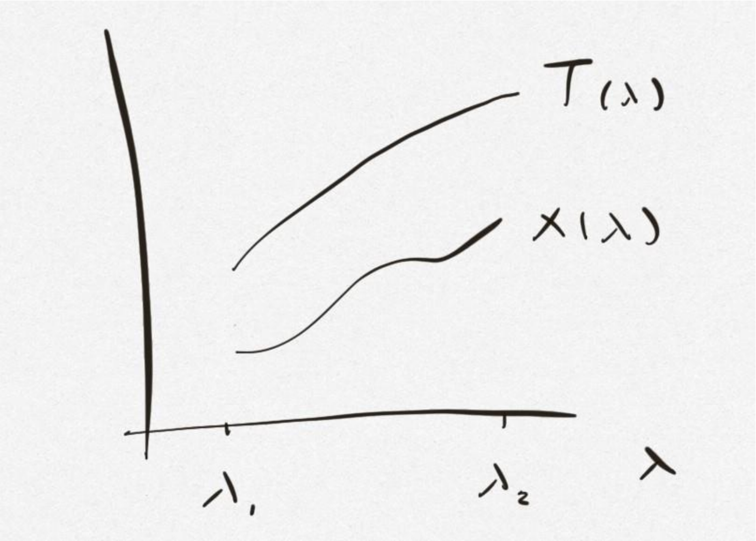
Newtonian Mechanics:

$$S[x(T)] = \int dT \left(\frac{m}{2} \left(\frac{dx}{dT} \right)^2 - U(x) \right)$$



Parameterized Particle Mechanics (PPM):

$$S[x(\lambda), T(\lambda)] = \int d\lambda \dot{T} \left(\frac{m}{2} \left(\frac{\dot{x}}{\dot{T}} \right)^2 - U(x(\lambda)) \right)$$



$$\dot{} = \frac{d}{d\lambda}$$

General variation of the action:

$$\delta S = \int d\lambda \left(\frac{d}{d\lambda} \left(\frac{m}{2} \frac{\dot{x}^2}{\dot{T}^2} + U \right) \delta T - \left(m \frac{d}{d\lambda} \left(\frac{\dot{x}}{\dot{T}} \right) + \dot{T} \frac{dU}{dx(\lambda)} \right) \delta x \right)$$
$$+ \left[- \left(\frac{m}{2} \frac{\dot{x}^2}{\dot{T}^2} + U \right) \delta T + m \frac{\dot{x}}{\dot{T}} \delta x \right]_{\lambda_1}^{\lambda_2}$$

Symmetries:

1. Reparameterization/diffeomorphism symmetry:

Action is invariant under $\lambda \to \tilde{\lambda} = f(\lambda)$ (with $\tilde{\lambda} = \lambda$ at the boundaries).
For infinitesimal changes, this corresponds to the following transformation of fields:

$$\delta T = \mathcal{L}_\xi T \qquad \delta x = \mathcal{L}_\xi x \qquad (\xi = \epsilon(\lambda)\partial_\lambda)$$

1. Reparameterization/diffeomorphism symmetry:

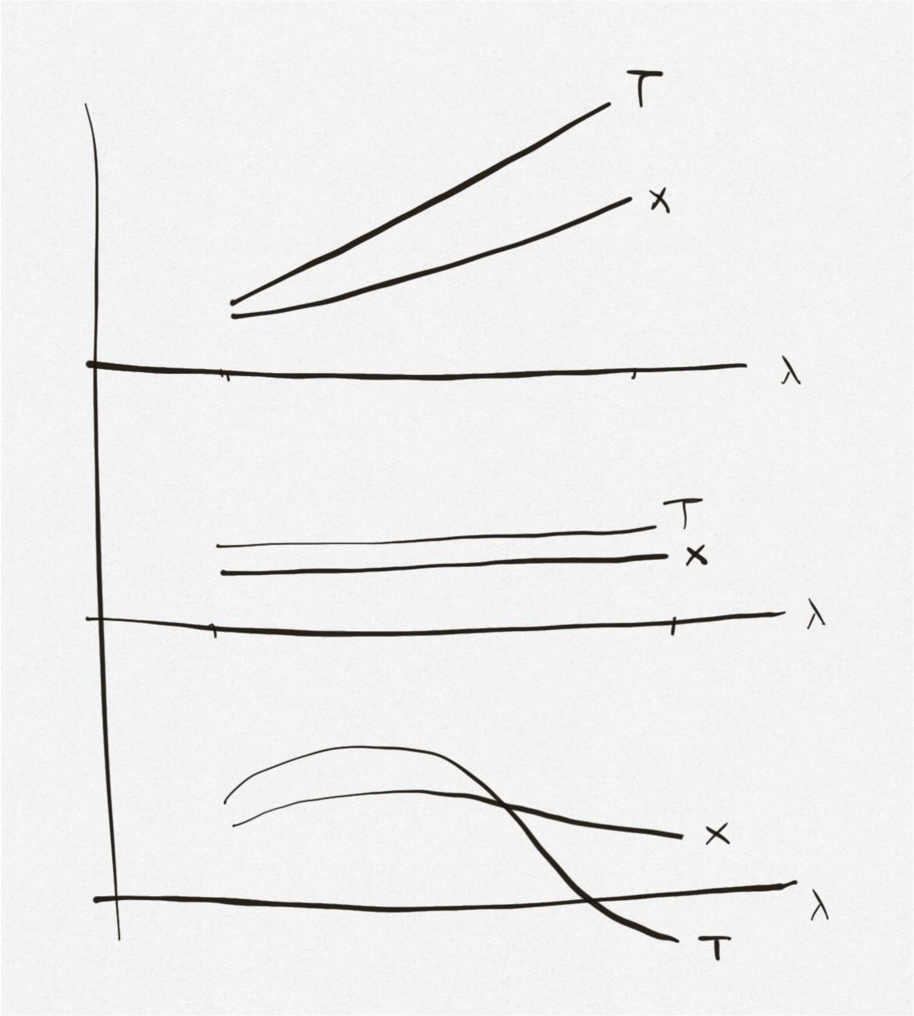
$$T(\lambda) \to T(\lambda) + \epsilon(\lambda)\frac{dT}{d\lambda} \qquad x(\lambda) \to x(\lambda) + \epsilon(\lambda)\frac{dx}{d\lambda}$$

$$\delta_\epsilon S = \int d\lambda \left(\frac{d}{d\lambda}\left(\frac{m}{2}\frac{\dot{x}^2}{\dot{T}^2} + U\right)\epsilon\frac{dT}{d\lambda} - \left(m\frac{d}{d\lambda}\left(\frac{\dot{x}}{\dot{T}}\right) + \dot{T}\frac{dU}{dx(\lambda)}\right)\epsilon\frac{dx}{d\lambda}\right)$$
$$+ \left[\epsilon\left(\frac{m}{2}\frac{\dot{x}^2}{\dot{T}} - \dot{T}U\right)\right]_{\lambda_1}^{\lambda_2}$$

$$T \overset{*}{=} \lambda$$

$$T \overset{*}{=} 0$$

$$T \overset{*}{=} \sin(\lambda)$$



$$0 = m\frac{d}{d\lambda}\left(\frac{\dot{x}}{\dot{T}}\right) + \dot{T}\frac{dU}{dx(\lambda)}$$

$$0 = \frac{d}{d\lambda}\left(\frac{m}{2}\frac{\dot{x}^2}{\dot{T}^2} + U\right)$$

<u>$T = \lambda$ Gauge</u>

$$0 = m\frac{d^2X}{dT^2} + \frac{dU}{dx(\lambda)}$$

$$0 = \frac{d}{dT}\left(\frac{m}{2}\left(\frac{dx}{dT}\right)^2 + U\right)$$

Symmetries:

2. Global time shift symmetry:

$$T(\lambda) \to T(\lambda) + C$$

$$\delta_C S = \int d\lambda \left(\frac{d}{d\lambda}\left(\frac{m}{2}\frac{\dot{x}^2}{\dot{T}^2} + U \right) C \right) + \left[-\left(\frac{m}{2}\frac{\dot{x}^2}{\dot{T}^2} + U \right) C \right]_{\lambda_1}^{\lambda_2}$$

So PPM is a bit more general than Newtonian Mechanics but contains its solutions and it formally has more dynamical symmetries: local diffeomorphism symmetry and global shift symmetry of the field $T(\lambda)$.

Path integral quantization of PPM gives identical results to standard quantum mechanics, independent of function $f(\lambda)$ in $T = f(\lambda)$ (see, e.g., Henneaux and Bunster).

Parameterized field theory (PFT):

Metrics:

$$g_{Newt} = -dT \otimes dT$$

$$g_{PPM} = -\left(\frac{dT}{d\lambda}\right)^2 d\lambda \otimes d\lambda$$

The generalization to higher dimensional manifolds parametrized/coordinatized by N $(\lambda^1, \lambda^2, \ldots \lambda^N)$:

$$g_M = -dT \otimes dT + \delta_{ij} dX^i \otimes dX^j$$

$$g_{M(PFT)} = -\frac{\partial T}{\partial \lambda^\mu}\frac{\partial T}{\partial \lambda^\nu} d\lambda^\mu \otimes d\lambda^\nu + \delta_{ij}\frac{\partial X^i}{\partial \lambda^\mu}\frac{\partial X^j}{\partial \lambda^\nu} d\lambda^\mu \otimes d\lambda^\nu$$

Minkowski PFT

$$X^I = (T, X^i)$$

$$g_{\mu\nu} = \eta_{IJ}\partial_\mu X^I \partial_\nu X^J \qquad (\eta_{IJ} = (-1,1,1,1))$$

For example, for a scalar field:

$$S[\phi, X^I] = \int d^4\lambda \sqrt{-g}\left(-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - U(\phi)\right)$$

General variation of the action:

$$\delta S = \int d^4\lambda \Bigg(\big(\partial_\beta \big(\sqrt{-g} g^{\alpha\beta}\partial_\alpha \phi\big) - \sqrt{-g}U'(\phi)\big)\delta\phi - \partial_\beta \big(\sqrt{-g} T^{\alpha\beta}\eta_{IJ}\partial_\alpha X^I\big)\delta X^J$$
$$+ \partial_\beta \Big(- \sqrt{-g} g^{\alpha\beta}\partial_\alpha \phi \delta\phi + \sqrt{-g} T^{\alpha\beta}\eta_{IJ}\partial_\alpha X^I \delta X^J \Big)\Bigg)$$

where

$$T^{\alpha\beta} = \big(-\frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - U(\phi)\big) g^{\alpha\beta} + \partial^\alpha\phi\partial^\beta\phi$$

General variation of the action:

$$\delta S = \int d^4\lambda \Big(\big(\partial_\beta(\sqrt{-g} g^{\alpha\beta}\partial_\alpha\phi) - \sqrt{-g}U'(\phi)\big)\delta\phi - \partial_\beta(\sqrt{-g}T^{\alpha\beta}\eta_{IJ}\partial_\alpha X^I)\delta X^J$$
$$+ \partial_\beta\Big(- \sqrt{-g} g^{\alpha\beta}\partial_\alpha\phi\delta\phi + \sqrt{-g}T^{\alpha\beta}\eta_{IJ}\partial_\alpha X^I \delta X^J\Big)\Big)$$

$$T^{\alpha\beta} = \Big(- \frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - U(\phi)\Big) g^{\alpha\beta} + \partial^\alpha\phi\partial^\beta\phi$$

Symmetries:

1. Spacetime diffeomorphisms:

Action is invariant under $\lambda^\mu \to \tilde{\lambda}^\mu(\lambda)$ (with $\tilde{\lambda}^\mu = \lambda^\mu$ at the boundaries)

$$X^I \to X^I + \xi^\mu\partial_\mu X^I \qquad \phi \to \phi + \xi^\mu\partial_\mu\phi \quad \Rightarrow \quad L \to L + \mathcal{L}_\xi L$$

$$\delta S_\xi = \int d^4\lambda \partial_\beta(\xi^\beta L)$$

There exist solutions where $\partial_\alpha X^I = \delta^I_\alpha$ and the equations of motion become:

$$0 = \partial_J \partial^J \phi - U'(\phi)$$

$$0 = \partial_J T^{IJ}$$

2. Global Poincare transformations:

$$X^I \to \Lambda^I{}_J X^J + P^I \qquad \phi \to \phi$$

$$\delta g_{\mu\nu} = \delta(\eta_{IJ} \partial_\mu X^I \partial_\nu X^J) = 0 \quad \Rightarrow \quad \delta S_{\Lambda,P} = 0$$

$$0 \overset{EOM}{=} \int d^4\lambda \left(\partial_\beta \left(\sqrt{-g} T^{\alpha\beta} \partial_\alpha X_{[I} X_{J]} \right) J^{IJ} + \partial_\beta \left(\sqrt{-g} T^{\alpha\beta} \partial_\alpha X_I \right) P^I \right)$$

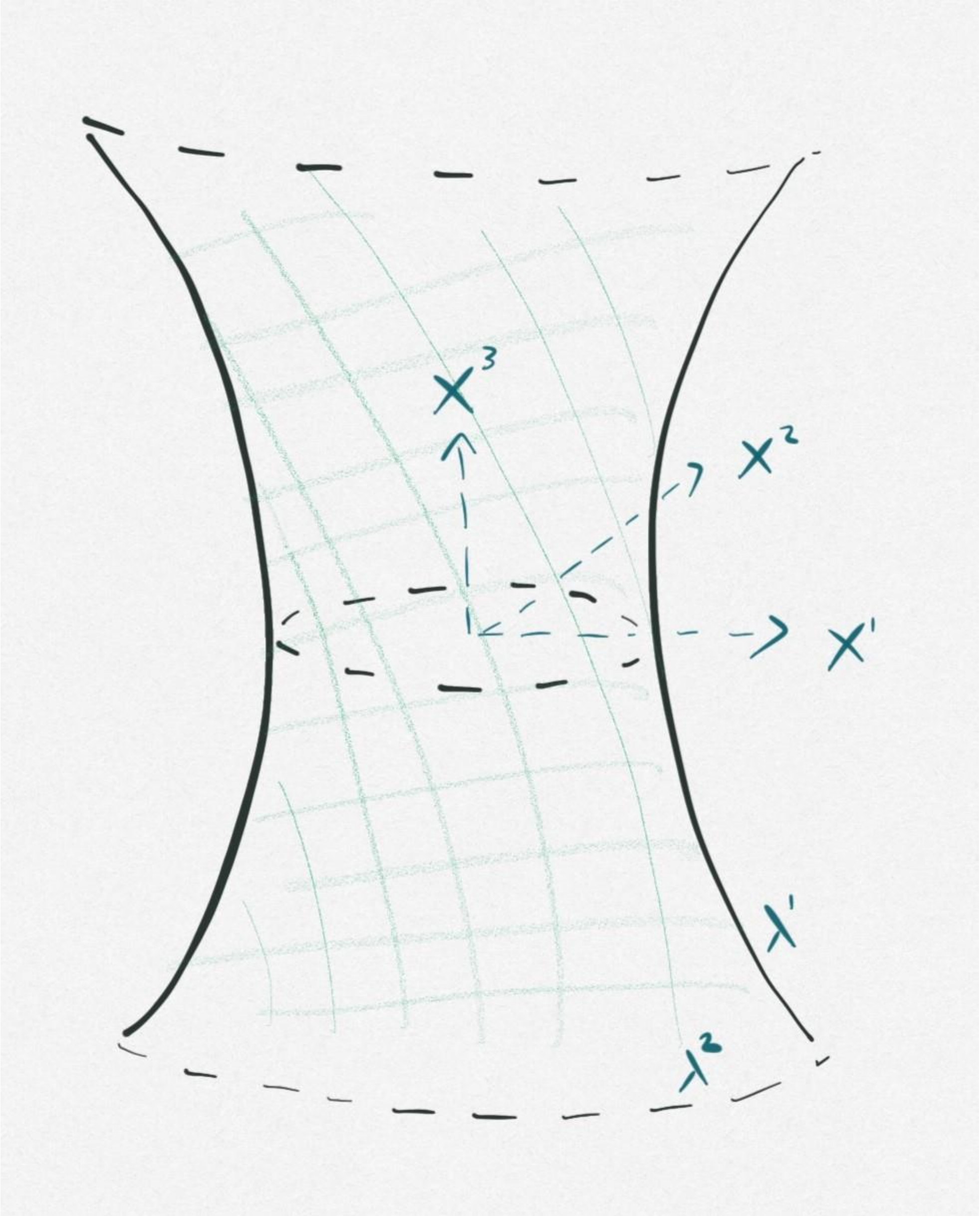
De Sitter PFT

Is there a PFT formulation of field theory in de Sitter space? Consider $\boldsymbol{DS^4}$ as a surface embedded in $\mathbb{R}^{(1,4)}$ with Cartesian/inertial coordinates
$\boldsymbol{X^A}$:

$$L^2 = \eta_{AB} X^A X^B$$
$$(\eta_{AB} = \mathrm{diag}(-1,1,1,1,1))$$

The metric on $\boldsymbol{DS^4}$ coordinatized by $\boldsymbol{\lambda^\mu}$ is then given by:

$$g_{\mu\nu} = \eta_{AB} \partial_\mu X^A \partial_\nu X^B$$



$$S[\phi, X^A, W] = \int d^4\lambda \sqrt{-g}\Big(-\frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - U(\phi) - W(\eta_{AB}X^A X^B - L^2)\Big)$$

Symmetries:

1. Spacetime diffeomorphisms:

Just as in the Minkowski case, the ction is invariant under $\lambda^\mu \to \tilde{\lambda}^\mu(\lambda)$ (with $\tilde{\lambda}^\mu = \lambda^\mu$ at the boundaries)

$$X^A \to X^A + \mathcal{L}_\xi X^A \qquad \phi \to \phi + \mathcal{L}_\xi \phi \qquad W \to W + \mathcal{L}_\xi W \Rightarrow L \to L + \mathcal{L}_\xi L$$

$$\delta S_\xi = \int d^4\lambda \partial_\beta(\xi^\beta L)$$

2. Global De sitter group ($SO(1,4)$) transformations:

$$\delta X^A = J^A{}_B X^B \qquad J^{AB} = -J^{AB}$$

$$\delta g_{\mu\nu} = \delta(\eta_{AB}\partial_\mu X^A \partial_\nu X^B) = 0 \quad \Longrightarrow \quad \delta_V S = 0$$

$$0 \overset{EOM}{=} \int d^4\lambda \partial_\beta\left(\sqrt{-g}\mathcal{T}^{\alpha\beta}\xi_\alpha\right) \qquad \xi_\alpha = (X_B J^{AB}\partial_\alpha X_A)$$

$$\mathcal{T}^{\alpha\beta} = \left(\left(-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - U(\phi) - \lambda(\eta_{AB}X^A X^B - L^2)\right)g^{\alpha\beta} + \partial^\alpha\phi\partial^\beta\phi\right)$$

The ξ_α are the Killing vectors of $\mathbb{R}^{(1,4)}$ tangent to DS^4
i.e. are the killing vectors of DS^4.

As in the previous cases, solutions for $X^A(\lambda^\mu)$ exist so that familiar equations of motion for a scalar field in DS^4 are recovered.

PFTs are non-gravitational field theories which nonetheless have the spacetime diffeomorphism symmetry closely associated with gravity. This seems like a promising starting point for including extra structure to incorporate gravitational dynamics.

For comparison, we will first look at the standard special-relativistic description of matter fields, see what symmetries are present there, and how this motivated Tom Kibble's recovery of general relativity as a theory of gauged symmetries.

Field Theory in Minkowski Space

$$S[\phi] = -\frac{1}{2}\int d^4x\sqrt{-\eta}(\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + U(\phi))$$

Action is diffeomorphism covariant due to it being built from tensor fields but this is not generally a symmetry as not all these fields are dynamical.

$$\phi \to \phi + \mathcal{L}_\xi\phi \qquad \eta_{\mu\nu} \to \eta_{\mu\nu} + \mathcal{L}_\xi\eta_{\mu\nu}$$

Under only

$$\phi \to \phi + \mathcal{L}_\xi\phi$$

$$\delta_\xi S = -\int d^4x\sqrt{-g}T^{\mu\nu}\nabla_{(\mu}\xi_{\nu)} + \int_{\partial M}\ell$$

Therefore there *is* a symmetry corresponding to diffeomorphisms generated by the ten Killing vectors of Minkowski space ($\nabla_{(\mu}\xi_{\nu)} = 0$) and this implies the conservation of energy-momentum and conserved quantities associated with coordinate Lorentz transformations.

$$\delta S_\xi \overset{EOM}{=} -\int d^4x \partial_\alpha \left(\sqrt{-\eta} T^{\alpha\beta} \xi_\beta\right)$$

Global Lorentz symmetry:

To write actions for fermions we need to introduce a non-dynamical object $\bar{e}_\mu^I$ called the Minkowski co-tetrad:

$$\eta_{\mu\nu} = \eta_{IJ} \bar{e}_\mu^I \bar{e}_\nu^J$$

e.g. Action for a Weyl fermion, potentially coupled to other gauge fields:

$$S[\chi] = i \int d^4x \sqrt{-\eta} \bar{e}_I^\mu \chi^a \sigma^I_{aa'} \mathcal{D}_\mu \chi^{a'}$$

$$S[\chi] = i \int d^4x \sqrt{-\eta}\, e_I^{\mu} \chi^a \sigma^I_{aa'} \mathcal{D}_{\mu} \chi^{a'}$$

Now under an $SL(2, C)$ transformation of the spinor (an 'internal' transformation, coordinates are not change):

$$\chi^a \to L^a{}_b \chi^b \qquad (\Lambda^I{}_J = \frac{1}{2} \sigma^I_{aa'} (L^*)^{a'}{}_{b'} \bar{\sigma}_J^{b'b} L^a{}_b)$$

$$\bar{e}_I^{\mu} \to (\Lambda^{-1})^J{}_I \bar{e}_I^{\mu}$$

$$\delta_L S[\chi] = 0$$

Bosonic fields couple to the Lorentz-invariant $\eta_{\mu\nu}$ hence the Lagrangians of the standard model have a global Lorentz symmetry alongside the limited diffeomorphism symmetry.

Gauging global symmetry

Kibble (1961) proposed promoting the global Lorentz symmetry to a local one.

$$L^a{}_b \to L^a{}_b(x) \qquad (\Lambda^I{}_J \to \Lambda^I{}_J(x))$$

It is then necessary to introduce a connection/gauge field so that a covariant derivative can be constructed:

$$D^{(\omega)}_\mu \chi^{a'} \equiv \mathcal{D}_\mu \chi^{a'} + \omega^{a'}{}_{b'\mu}\chi^{b'}$$

$$D^{(\omega)}_\mu (L^{a'}{}_{b'}(x)\chi^{b'}) = L^{a'}{}_{b'} D^{(\omega)}_\mu \chi^{b'}$$

$$\omega^I{}_{J\mu} \to \Lambda^I{}_K \omega^K{}_{L\mu}(\Lambda^{-1})^L{}_J - \partial_\mu \Lambda^I{}_K (\Lambda^{-1})^K{}_J$$

Hence the action

$$S[\chi,\omega] = i\int d^4x\sqrt{-\eta}e^{\mu}_{I}\chi^{a}\sigma^{I}_{aa'}D^{(\omega)}_{\mu}\chi^{a'}$$

Possesses a local gauge symmetry under $\boldsymbol{SL(2,C)}$ transformations, though the metric is still Minkowskian. We are now obliged to introduce, additionally, an action involving $\boldsymbol{\omega_{\mu}^{IJ}}$ otherwise its equation of motion from the above action puts artificial constraints on the fermions.

Additionally, the prior geometry can be removed by promoting $\boldsymbol{e_{\mu}^{I}}$ to a dynamical field, simultaneously promoting the limited diffeomorphism symmetry to full diffeomorphism symmetry. Again, an additional action involving $\boldsymbol{e_{\mu}^{I}}$ should be introduced so an action such as the above with $\boldsymbol{\bar{e}_{\mu}^{I}} \rightarrow \boldsymbol{e_{\mu}^{I}}$ doesn't produce equations of motion that artificially constrain the matter.

$$g_{\mu\nu} = \eta_{IJ}e^{I}_{\mu}e^{J}_{\nu}$$

A simple possibility is the following one, which contains derivatives of $\boldsymbol{\omega_\mu^{IJ}}$ but at the lowest order (linear):

$$S_g[e,\omega] = \frac{1}{64\pi G}\int d^4x\varepsilon^{\mu\nu\alpha\beta}\epsilon_{IJKL}e^I_\mu e^J_\nu R^{KL}{}_{\alpha\beta}(\omega)$$

$$R^{IJ}{}_{\mu\nu} \equiv 2\partial_{[\mu}\omega^{IJ}{}_{\nu]} + 2\omega^I{}_{K[\mu}\omega^{KJ}{}_{\nu]}$$

For a combined gravity + matter system $S_g[e,\omega] + S_M[e,\omega,\chi,\dots]$

$$\frac{\delta S_g}{\delta\omega_\mu^{IJ}} = -\frac{\delta S_M}{\delta\omega_\mu^{IJ}} \quad \Rightarrow \quad \omega_\mu^{IJ} = \omega_\mu^{IJ}(e,\partial e,\chi)$$

$\Downarrow$

$$\frac{\delta S_g}{\delta e^I_\mu} = -\frac{\delta S_M}{\delta e^I_\mu}$$

Einstein's Equations!

Einstein-Cartan-Sciama-Kibble case

- Spacetime structure: $\bar{e}^I_\mu \to \eta_{\mu\nu}$

- Global Lorentz symmetry and limited diffeomorphism symmetry (Killing vectors of Minkowski space)

$$+\omega^I{}_{J\mu}$$

- Local Lorentz symmetry and limited diffeomorphism symmetry

$$\bar{e}^I_\mu \to e^I_\mu, +S_g(e,\omega)$$

- Local Lorentz symmetry and full diffeomorphism symmetry, and gravitation

PFT case

- Spacetime structure without gravity: X^I
- Global Poincare symmetry and full diffeomorphism symmetry

$+?$

Gauging the global symmetries of Minkowski PFT

$$X^I \to \Lambda^I{}_J(x) X^J + P^I(x)$$

$$D^{(\mathcal{P})}_\mu X^I \equiv \partial_\mu X^I + \omega^I{}_{J\mu} X^J + \theta^I_\mu \quad \Longrightarrow \quad D^{(\mathcal{P})}_\mu X^I \to \Lambda^I{}_J D^{(\mathcal{P})}_\mu X^J$$

If:

$$\omega^I{}_{J\mu} \to \Lambda^I{}_K \omega^K{}_{L\mu} (\Lambda^{-1})^L{}_J - \partial_\mu \Lambda^I{}_K (\Lambda^{-1})^K{}_J$$
$$\theta^I_\mu \to \Lambda^I{}_J \theta^J_\mu - \partial_\mu P^I$$

(Grignani and Nardelli, 'Gravity and the Poincare group', PRD 1992)

Einstein-Cartan-Sciama-Kibble case:

$$S_g[e,\omega] = \frac{1}{64\pi G}\int d^4x \varepsilon^{\mu\nu\alpha\beta}\epsilon_{IJKL} e^I_\mu e^J_\nu R^{KL}{}_{\alpha\beta}(\omega)$$

Gauged PFT case:

$$S_g[X,\theta,\omega] = \frac{1}{64\pi G}\int d^4x \varepsilon^{\mu\nu\alpha\beta}\epsilon_{IJKL} D^{(\mathcal{P})}_\mu X^I D^{(\mathcal{P})}_\nu X^J R^{KL}{}_{\alpha\beta}(\omega)$$

Though a gauge transformation with $\boldsymbol{P^I = -X^I}$ gives $\boldsymbol{X^I = 0}$ in that gauge and

$$S_g[\theta,\omega] \stackrel{*}{=} \frac{1}{64\pi G}\int d^4x \varepsilon^{\mu\nu\alpha\beta}\epsilon_{IJKL} \theta^I_\mu \theta^\nu_J R^{KL}{}_{\alpha\beta}(\omega)$$

More general actions can be considered (e.g. quadratic in curvature) leading to more propagating degrees of freedom *(Poincare gauge theory).*

Gauging the global symmetries of De Sitter PFT

The story is very similar in the case of De Sitter PFT, with global De Sitter group invariance promoted to a local one:

$$X^A \to M^A{}_B(x) X^B \qquad\qquad (M^A{}_B \in SO(1,4))$$

$$D_\mu X^A = \partial_\mu X^A + A^A{}_{B\mu} X^B$$

This gauging doesn't affect the constraint

$$X^A X_A - L^2 = 0$$

and so a useful gauge is

$$X^A \overset{*}{=} L\delta^A_4$$

$$S[A, X, \lambda] = -\frac{\alpha}{4}\int d^4x\Big(\epsilon_{ABCDE}\epsilon^{\alpha\beta\mu\nu}X^E F^{AB}{}_{\alpha\beta}F^{CD}{}_{\mu\nu} - \lambda(\eta_{AB}X^A X^B - L^2)\Big)$$

(Stelle, West 'De Sitter Invariance and the Geometry of the Einstein-Cartan theory' (1979).)

$$X^A \stackrel{*}{=} L\delta^A_4 \qquad A^{AB}{}_{\mu} \stackrel{*}{=} \begin{pmatrix} 0 & \frac{1}{L}e^I_\mu \\ -\frac{1}{L}e^I_\mu & \omega^{IJ}{}_\mu \end{pmatrix}$$

$$S[e,\omega] \stackrel{*}{=} \int d^4x \epsilon_{IJKL}\varepsilon^{\alpha\beta\mu\nu}\Big(\frac{\alpha}{L^2}\Big(e^I_\alpha e^J_\beta R^{KL}{}_{\mu\nu}(\omega) - \frac{1}{L^2}e^I_\alpha e^J_\beta e^K_\mu e^L_\nu\Big) - \alpha R^{IJ}{}_{\alpha\beta}(\omega)R^{KL}{}_{\mu\nu}(\omega)\Big)$$

The gauge fixed action is the action for MacDowell-Mansouri gravity.

PFT case:

- Spacetime structure: X^I

- Global Poincare symmetry or de Sitter symmetry and full diffeomorphism symmetry

$$+(\omega^{IJ}{}_{\mu}, \theta^I_\mu), +S[X, e, \omega] \qquad \text{(Poincare case)}$$

$$+A^{AB}{}_{\mu}, +S[X, A] \qquad \text{(de Sitter case)}$$

- Local Poincare symmetry or de Sitter symmetry and full diffeomorphism symmetry and gravitation.

Gauging *some* of the global symmetries of Minkowski PFT (TZ, Urban, Marzola, Koivisto 2018)

What if *only* the global Lorentz symmetry of Minkowski PFT was gauged?
Can something with a General-Relativistic limit emerge?

$$X^I \to \Lambda^I{}_J(x) X^J$$

$$D_\mu^{(\omega)} X^I \equiv \partial_\mu X^I + \omega^I{}_{J\mu} X^J$$

$$(D_\mu^{(\omega)} X^I \to \Lambda^I{}_J D_\mu^{(\omega)} X^J)$$ if $\boldsymbol{\omega}_\mu^{IJ}$ transforms as a connection

$$g_{\mu\nu} = \eta_{IJ} D_\mu^{(\omega)} X^I D_\nu^{(\omega)} X^J$$

$$S_\gamma[X,\omega] = \frac{1}{2}\int d^4x\, \tilde{\epsilon}^{\mu\nu\alpha\beta}\big(\epsilon_{IJKL} + \frac{2}{\gamma}\eta_{K[I}\eta_{J]L}\big) D^{(\omega)}_\mu X^I D^{(\omega)}_\nu X^J R^{KL}{}_{\alpha\beta}(\omega)$$

- Minkowski space solution $g_{\mu\nu} = \eta_{\mu\nu}$ a solution for any value of γ. For example $\omega_\mu^{IJ} = 0$ and $\partial_\mu X^I = \delta_\mu^I$.
- The action for tensor mode perturbations takes a simple form in upper Milne wedge with $\mathrm{T} \equiv (-X_I X^I)^{\frac{1}{2}}$ as a time coordinate:

$$S_{HH} = 2\int dT d^3x a^3 \Bigg(-\frac{1}{\gamma^2}\dot{H}_{ij}\dot{H}^{ij} - \frac{1}{a^2}h^{ab}\mathcal{D}_a H^{ij}\mathcal{D}_b H_{ij} - \frac{2k}{a^2}H_{ij}H^{ij}\Bigg)$$

Matches GR for $\gamma = \pm i$!

- FRW symmetry, parts of $\boldsymbol{\omega}$ can be eliminated via EOMs to give:

$$S[a(\lambda),T(\lambda)] \overset{b}{\propto} \int d\lambda a^3 \dot{T}\left(3\left(\frac{1}{\gamma^2}\frac{1}{a^2}\frac{\dot{a}^2}{\dot{T}^2}+\frac{k}{a^2}\right)+\frac{(1+\gamma^2)}{\gamma^2}\frac{1}{T^2}\right)$$

- Matches kinetic structure of GR for $\boldsymbol{\gamma = \pm i}$ (in the gauge $\boldsymbol{T = \lambda}$).

- The system is PPM! Choosing one of the two special values of $\boldsymbol{\gamma}$, the conserved Noether charge associated with the global $\boldsymbol{T(\lambda) \to T(\lambda) + C}$ shift symmetry is:

$$P_T = \frac{3}{8\pi G}\left(\frac{1}{a^2}\frac{\dot{a}^2}{\dot{T}^2}+\frac{k}{a^2}\right)a^3$$

- Note this differs significantly from Einstein-Cartan gravity where the corresponding γ term in the action (the Holst term) doesn't affect the propagation speed of gravity waves etc.

- Non-perturbative Hamiltonian analysis of model (Nikjoo and TZ, 2023) confirms that theory generally is equivalent to pressureless dust coupled to Ashtekar's chiral formulation of General Relativity e.g. if T acts as time coordinate:

$$\omega^{IJ}{}_{\mu} \stackrel{*}{=} \begin{pmatrix} 0 & \frac{1}{T}E^i_a \\ -\frac{1}{T}E^i_a & \Gamma^{ij}{}_a - i\epsilon^{ij}{}_k(K^k_a - \frac{1}{T}E^k_a) \end{pmatrix}$$

- Can this model provide a candidate for some of the dark matter or 'solve' the problem of time in quantum gravity? Unclear

Conclusions

- PFTs are viable generally-covariant alternatives to field theories in fixed backgrounds.

- Gauging their global symmetries leads quite straightforwardly to the inclusion of the gravitational interaction.

- Gauging selected global symmetries can lead to extensions to General Relativity or alternative formulations of it (for example if only translations are gauged, then teleparallel theories result).

- It is likely that the gauging of PFTs with other global symmetries lead to further novel extensions to General Relativity.